

# **CP Pre-Calculus Curriculum Maps**

**[Unit 1: Essential Skills For Pre-Calculus](#)**

**[Unit 2: Relations, Functions and Graphs](#)**

**[Unit 3: The Trigonometric Function](#)**

**[Unit 4: Graphs of Trigonometric Functions](#)**

**[Unit 5: Trigonometric Identities and Equations](#)**

**[Unit 6: Exponential and Logarithmic Functions](#)**

<b>Grade: 11 - 12</b> <b>Subject: CP</b> <b>Pre-Calculus</b>	<b>Unit 1: Essential Skills For Pre-Calculus</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: Functions, Relations and Operations with Them</p> <p>Rationale: Strong skills are essential for success in Pre-Calculus and Calculus. Reviewing key mathematical processes will help on the path to future success.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<p>Pre-Calculus connects the many pieces of mathematics from your past learning.</p> <p>Pre-Calculus is fundamental in the study of mathematics and science.</p>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>● What is Pre-Calculus?</li> <li>● How does the study of Pre-Calculus prepare you for Calculus?</li> <li>● Mathematically speaking, how does what you learned today connect to what you learned yesterday and what you will learn tomorrow?</li> <li>● How are functions and their graphs related?</li> <li>● How can technology be used to investigate properties of families of functions and their graphs?</li> <li>● How does explaining a process help me to better understand the idea?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know....</i></p> <p><b>Key Terms</b> - Relation, domain, range, function, vertical line test, function notation, operations with functions, composition of functions, slope-intercept form, point-slope form, standard form, coinciding, parallel and perpendicular lines, scatter plots, best-fit line, prediction equation, correlation coefficient, piecewise function, step function, greatest integer function, absolute value function, linear equation/inequality, system of equations/inequalities, linear programming consistent, inconsistent, dependent, independent, ordered triple, matrix operations, determinants, identity matrix, inverse matrix, symmetry, parent graphs/families, inverse functions and relations, continuity, end behavior, monotonicity, increasing, decreasing and constant functions, critical points and extrema, asymptotes, direct/inverse/joint variation</p> <p><i>Students will be able to ...</i></p> <ul style="list-style-type: none"> <li>● Determine whether a given relation is a function and perform operations with functions</li> <li>● Evaluate and find zeros of linear functions using functions notation</li> <li>● Graph and write functions and inequalities</li> <li>● Write equations of parallel and perpendicular lines</li> <li>● Model data using scatter plots and write prediction equations</li> <li>● Solve systems of equations and inequalities</li> <li>● Use linear programming to solve problems</li> <li>● Graph functions, relations, inverses, and inequalities</li> </ul>

	<ul style="list-style-type: none"> <li>● Analyze families of graphs</li> <li>● Investigate symmetry, continuity, end behavior, and transformations of graphs</li> <li>● Find asymptotes and extrema of functions</li> <li>● Solve problems involving direct, inverse, and joint variation</li> </ul>
<p><b>NJSLS Standards</b></p>	<p><b>A.APR.D.6.</b> Rewrite simple rational expressions in different forms; write <math>\frac{a(x)}{b(x)}</math> in the form <math>q(x) + \frac{r(x)}{b(x)}</math>, where <math>a(x)</math>, <math>b(x)</math>, <math>q(x)</math>, and <math>r(x)</math> are polynomials with the degree of <math>r(x)</math> less than the degree of <math>b(x)</math>, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p><b>A.CED.A.1.</b> Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p><b>A.CED.A.2.</b> Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p><b>A.CED.A.3.</b> Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</i></p> <p><b>A.REI.A.2.</b> Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> <p><b>A.REI.C.5.</b> Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.</p> <p><b>A.REI.C.6.</b> Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p><b>A.REI.C.8.</b> (+) Represent a system of linear equations as a single matrix equation in a vector variable.</p> <p><b>A.REI.C.9.</b> (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension <math>3 \times 3</math> or greater).</p> <p><b>A.REI.D.10.</b> Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).</p> <p><b>A.REI.D.12.</b> Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p> <p><b>F.IF.A.11.</b> Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If <math>f</math> is a function and <math>x</math> is an element of its domain,</p>

then  $f(x)$  denotes the output of  $f$  corresponding to the input  $x$ . The graph of  $f$  is the graph of the equation  $y = f(x)$ .

**F.IF.A.2.** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

**F.IF.B.4.** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★

**F.IF.B.5.** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function  $h(n)$  gives the number of person-hours it takes to assemble  $n$  engines in a factory, then the positive integers would be an appropriate domain for the function.*★

**F.IF.B.6.** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

**F.IF.C.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
- b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

**F.BF.A.1.** Write a function that describes a relationship between two quantities.★

- c. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*

**F.BF.B.3.** Identify the effect on the graph of replacing  $f(x)$  by  $f(x) + k$ ,  $kf(x)$ ,  $f(kx)$ , and  $f(x + k)$  for specific values of  $k$  (both positive and negative); find the value of  $k$  given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

**F.BF.B.4.** Find inverse functions.

- a. Solve an equation of the form  $f(x) = c$  for a simple function  $f$  that has an inverse and write an expression for the inverse. *For example,  $f(x) = 2x^3$  or  $f(x) = (x+1)/(x-1)$  for  $x \neq 1$ .*
- b. (+) Verify by composition that one function is the inverse of another.
- c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

	<b>F.LE.B.5.</b> Interpret the parameters in a linear or exponential function in terms of a context.	
<b>21<sup>st</sup> Century Themes</b> Global Awareness Financial and Economic Literacy Health Literacy Civic Literacy		<b>21<sup>st</sup> Century Skills</b> Critical Thinking and Problem Solving Communication and Collaboration
<b>Materials and Resources</b>	<ul style="list-style-type: none"> <li>● Glencoe Advanced Mathematical Concepts – Precalculus with Applications</li> <li>● Graphing Calculators</li> <li>● Document Camera</li> </ul>	
<b>Notes</b>		

<b>Grade: 11 - 12</b> <b>Subject: CP</b> <b>Pre-Calculus</b>	<b>Unit 2: Relations, Functions and Graphs</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: Polynomial and Rational Functions</p> <p>Rationale: Everyday life abounds in apparently random phenomena: changing weather, traffic on the freeway, lightning paths, ocean turbulence and many others. This chaotic behavior is analyzed in mathematics.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>● Polynomial functions arise naturally in many applications.</li> <li>● Many complicated functions can be approximated by polynomial functions or their quotients – rational functions.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>● How are polynomial functions classified by degree?</li> <li>● What are the zeros of a polynomial function and how can we find them?</li> <li>● Explain how to use zeros and end behaviors to sketch a possible graph of a polynomial function.</li> <li>● What is a complex number and how do you perform operations with complex numbers?</li> <li>● While solving quadratic equations by factoring, why is each factor set equal to zero?</li> <li>● What is the significance of <math>\pm</math> in the Quadratic Formula?</li> <li>● Explain how to sketch the graph of a quadratic function given in vertex form.</li> <li>● Explain how to sketch the graph of a quadratic function given in standard form.</li> <li>● What are the limitations of synthetic division?</li> <li>● How are the Remainder and Factor Theorems related?</li> <li>● Why is the Integral Root Theorem a corollary to the Rational Root Theorem?</li> <li>● Describe how you find an upper bound and a lower bound for the zeros of a polynomial functions?</li> <li>● How is synthetic division used to determine which consecutive integers the real zeros of a polynomial function are located?</li> <li>● Why must all solutions of a rational equation be checked?</li> <li>● Why is it necessary to check for extraneous solutions in radical equations?</li> <li>● Explain the difference between solving an equation with one radical and solving an equation with more than one radical.</li> <li>● Why is it important to recognize the shape of the graph of each type of polynomial function when solving problems using real world data?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – polynomial in one variable, polynomial equation, root, zero, multiplicity, imaginary numbers, complex numbers, pure imaginary numbers, Fundamental Theorem of Algebra, degree, leading coefficient,</p>

	<p>completing the square, quadratic formula, discriminant, complex conjugates, complex conjugate theorem, Remainder Theorem, synthetic division, Factor Theorem, depressed polynomial, binomial factor, polynomial long division, Rational Root Theorem, Integral Root Theorem, Descartes' Rule of Signs, location principle, upper bound theorem, lower bound theorem, rational equation/inequality, partial fractions, radical equation/inequality, extraneous solution</p> <p><i>Students will be able to ....</i></p> <ul style="list-style-type: none"> <li>● Determine roots of polynomial equations</li> <li>● Add, Subtract, multiply and divide complex numbers</li> <li>● Solve quadratic, rational, and radical equations and inequalities</li> <li>● Find the factors of polynomials</li> <li>● Approximate real zeros of polynomial functions</li> <li>● Write and interpret polynomial functions that model real world data</li> </ul>
<p><b>NJSLS Standards</b></p>	<p><b>N.CN.A.1.</b> Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math> real.</p> <p><b>N.CN.A.2.</b> Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p> <p><b>N.CN.A.3.</b> (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.</p> <p><b>N.CN.B.4.</b> (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.</p> <p><b>N.CN.B.5.</b> (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. <i>For example, <math>(-1 + \sqrt{3}i)^3 = 8</math> because <math>(-1 + \sqrt{3}i)</math> has modulus 2 and argument <math>120^\circ</math>.</i></p> <p><b>N.CN.B.6.</b> (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.</p> <p><b>N.CN.C.7.</b> Solve quadratic equations with real coefficients that have complex solutions.</p> <p><b>N.CN.C.8.</b> (+) Extend polynomial identities to the complex numbers. <i>For example, rewrite <math>x^2 + 4</math> as <math>(x + 2i)(x - 2i)</math>.</i></p> <p><b>N.CN.C.9.</b> (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p> <p><b>A.SSE.A.1.</b> Interpret expressions that represent a quantity in terms of its context.★</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p>

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret  $P(1+r)^n$  as the product of  $P$  and a factor not depending on  $P$ .*

**A.SSE.A.2.** Use the structure of an expression to identify ways to rewrite it. *For example, see  $x^4 - y^4$  as  $(x^2)^2 - (y^2)^2$ , thus recognizing it as a difference of squares that can be factored as  $(x^2 - y^2)(x^2 + y^2)$ .*

**A.SSE.B.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

a. Factor a quadratic expression to reveal the zeros of the function it defines.

b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.

**A.APR.B.2.** Know and apply the Remainder Theorem: For a polynomial  $p(x)$  and a number  $a$ , the remainder on division by  $x - a$  is  $p(a)$ , so  $p(a) = 0$  if and only if  $(x - a)$  is a factor of  $p(x)$ .

**A.APR.B.3.** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

**A.REI.B.4.** Solve quadratic equations in one variable.

a. Use the method of completing the square to transform any quadratic equation in  $x$  into an equation of the form  $(x - p)^2 = q$  that has the same solutions. Derive the quadratic formula from this form.

b. Solve quadratic equations by inspection (e.g., for  $x^2 = 49$ ), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as  $a \pm bi$  for real numbers  $a$  and  $b$ .

**F.IF.C.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.

**F.IF.C.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

**21<sup>st</sup> Century Themes**  
 Global Awareness  
 Financial and Economic Literacy  
 Health Literacy  
 Civic Literacy

**21<sup>st</sup> Century Skills**  
 Critical Thinking and Problem Solving  
 Communication and Collaboration

**Materials and Resources**

- Glencoe Advanced Mathematical Concepts – Precalculus with Applications



	<ul style="list-style-type: none"><li>• Graphing Calculators</li><li>• Document Camera</li></ul>
<b>Notes</b>	

<b>Grade: 11<sup>th</sup> and 12<sup>th</sup></b> <b>Subject: CP</b> <b>Pre-Calculus</b>	<b>Unit 3: The Trigonometric Function</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: The Study of Trigonometry – Old and New</p> <p>Rationale: Students began their study of trigonometry in previous math courses with right triangles. They will extend their knowledge to understand how this information can help them solve a variety of real world problems.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>● Angles can be used to model and solve real world problems.</li> <li>● There exists a definite and distinct bridge between the fields of trigonometry and physics.</li> <li>● Trigonometry can be used to measure right triangles as well as oblique triangles.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>● Describe the difference between an angle with a positive measure and an angle with negative measure</li> <li>● What is a quick way to verify coterminal angles?</li> <li>● What are the reciprocal ratios of sine, cosine and tangent?</li> <li>● Explain the process of evaluating a trigonometric function using reference angles and the unit circle.</li> <li>● How do the values on the unit circle correlate to the rectangular graph of a trigonometric function?</li> <li>● Describe a way to use trigonometry to determine the height of the building where you live.</li> <li>● What is the Law of Sines and what is it used for?</li> <li>● What is the Law of Cosines and what is it used for?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know....</i></p> <p><b>Key Terms</b> – vertex, initial side, terminal side, standard position, degree, minutes, seconds, quadrantal angle, coterminal angle, reference angle, trigonometric ratios, cofunctions, unit circle, angle of elevation, angle of depression, inverse trigonometric functions, Law of Sines, Law of Cosines, ambiguous case, Area of a triangle using sine/cosine</p> <p><i>Students will be able to....</i></p> <ul style="list-style-type: none"> <li>● Convert decimal degree measures to degrees, minutes, seconds and vice versa</li> <li>● Identify angles that are coterminal with a given angle</li> <li>● Solve right triangles and non-right triangles</li> <li>● Find the values of trigonometric functions</li> <li>● Find the areas of triangle</li> </ul>
<b>NJSLS Standards</b>	<p><b>G.SRT.C.6.</b> Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p>

	<p><b>G.SRT.C.7.</b> Explain and use the relationship between the sine and cosine of complementary angles.</p> <p><b>G.SRT.C.8.</b> Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★</p> <p><b>G.SRT.D.9.</b> (+) Derive the formula <math>A = 1/2 ab \sin(C)</math> for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p><b>G.SRT.D.10.</b> (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p><b>G.SRT.D.11.</b> (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>
<p><b>21<sup>st</sup> Century Themes</b>  Financial and Economic Literacy  Health Literacy</p>	<p><b>21<sup>st</sup> Century Skills</b>  Critical Thinking and Problem Solving  Communication and Collaboration</p>
<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>● Glencoe Advanced Mathematical Concepts – Precalculus with Applications</li> <li>● Graphing Calculators</li> <li>● Document Camera</li> </ul>
<p><b>Notes</b></p>	

<b>Grade: 11 - 12</b> <b>Subject: CP</b> <b>Pre-Calculus</b>	<b>Unit 4: Graphs of Trigonometric Functions</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: Modeling Periodic Behavior</p> <p>Rationale: Many occurrences in the real world can be modeled with sinusoidal equations including the respiration of a person, the oceanic tides, the average monthly temperatures and a person's blood pressure. These functions can help us predict future behavior.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>● Angles are the domain elements of the trigonometric functions</li> <li>● The radian allows mathematicians to link between an angle's measure to the length of its radius.</li> <li>● Real world data can be modeled with a sinusoidal function.</li> <li>● Members within a family of functions, including the trigonometric functions have common characteristics</li> <li>● Function composition extends our ability to model periodic phenomena like heartbeats and sound waves.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>● What are radians and how are they related to degrees?</li> <li>● How are linear and angular velocities similar/different? That is, how can two people on a rotating carousel have the same angular velocity but different linear velocity?</li> <li>● Why are the trigonometric functions periodic?</li> <li>● What is the effect of A, B, h, and k on the graph of a Sine or Cosine curve <ul style="list-style-type: none"> <li style="text-align: center;"><math>y = A \sin B(x - h) + k</math></li> <li>using the equations <math>y = A \cos B(x - h) + k</math></li> </ul> </li> <li>● Why is it necessary to restrict the domain in order to discuss inverse trigonometric functions?</li> <li>● What does it mean to find an exact value of a trigonometric function?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – radian, circular arc, central angle, length of an arc, sector, area of a circular sector, angular displacement, angular velocity, dimensional analysis, linear velocity, periodic function, period, amplitude, phase shift, frequency, compound functions, sinusoidal function, principal values,</p> <p><i>Students will be able to....</i></p> <ul style="list-style-type: none"> <li>● Convert from radian measure to degree measure and vice versa</li> <li>● Find linear and angular velocity</li> <li>● Use and draw graphs of trigonometric functions and their inverses</li> <li>● Find the amplitude, the period, the phase shift, and the vertical shift for trigonometric functions</li> <li>● Write trigonometric equations to model a given situation</li> </ul>

<p><b>NJSLS Standards</b></p>	<p><b>F.IF.C.7.</b> Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p><b>F.BF.B.4.</b> Find inverse functions.</p> <p>d. (+) Produce an invertible function from a non-invertible function by restricting the domain.</p> <p><b>F.TF.A.1.</b> Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p><b>F.TF.A.2.</b> Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p> <p><b>F.TF.A.3.</b> (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosines, and tangent for <math>x</math>, <math>\pi + x</math>, and <math>2\pi - x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p> <p><b>F.TF.A.4.</b> (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p> <p><b>F.TF.B.5.</b> Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★</p> <p><b>F.TF.B.6.</b> (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.</p> <p><b>F.TF.C.8.</b> Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to find <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> given <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> and the quadrant of the angle.</p> <p><b>G.C.B.5.</b> Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.</p>
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<p><b>21<sup>st</sup> Century Themes</b> Global Awareness Financial and Economic Literacy</p>	<p><b>21<sup>st</sup> Century Skills</b> Critical Thinking and Problem Solving Creativity and Innovation Communication and Collaboration</p>
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<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>● Glencoe Advanced Mathematical Concepts – Precalculus with Applications</li> <li>● Graphing Calculators</li> <li>● Document Camera</li> </ul>
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<p><b>Notes</b></p>	
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<b>Grade: 11 - 12</b> <b>Subject: CP</b> <b>Pre-Calculus</b>	<b>Unit 5: Trigonometric Identities and Equations</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: Solving Puzzles - Connections among the Trigonometric Functions</p> <p>Rationale: Appreciation for the interlocking patterns that can be woven from the six basic trigonometric functions will take on greater meaning when viewed through the eyes of a calculus student.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>● Identities are important when working with trigonometric functions in calculus.</li> <li>● Proving identities gives insight into the way mathematical proofs are constructed</li> <li>● Equivalent expressions can be written in a variety of formats</li> <li>● Sinusoidal functions have an infinite number of solutions that are the same interval apart.</li> <li>● Models with trigonometric functions embrace the periodic rhythms of our world.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>● What does it mean to prove an identity?</li> <li>● What does it mean when we solve a trigonometric equation?</li> <li>● How do we represent multiple solutions on a given domain?</li> <li>● How can we use identities to simplify the process involved in solving an equation?</li> <li>● How can you use the sum and difference identities to find values for the secant, cosecant and cotangent functions of a sum or difference?</li> <li>● Why do many trigonometric equations have infinitely many solutions?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – identity, trigonometric identities, reciprocal identities, quotient identities, Pythagorean identities, symmetry identities, opposite-angle identities, sum and difference identities, double angle identities, half-angle identities, principal values, normal form, normal line, distance from a point to a line, relative position of the origin</p> <p><i>Students will be able to...</i></p> <ul style="list-style-type: none"> <li>● Use reciprocal, quotient, Pythagorean, symmetry, and opposite angle identities</li> <li>● Verify trigonometric identities</li> <li>● Use sum, difference, double angle, and half angle identities</li> <li>● Solve trigonometric equations and inequalities</li> <li>● Find the distance between a point and a line and the distance between two lines</li> </ul>

<p><b>NJSLS Standards</b></p>	<p><b>F.TF.A.3.</b> (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for <math>\pi/3</math>, <math>\pi/4</math> and <math>\pi/6</math>, and use the unit circle to express the values of sine, cosines, and tangent for <math>x</math>, <math>\pi + x</math>, and <math>2\pi - x</math> in terms of their values for <math>x</math>, where <math>x</math> is any real number.</p> <p><b>F.TF.B.7.</b> (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. ★</p> <p><b>F.TF.C.8.</b> Prove the Pythagorean identity <math>\sin^2(\theta) + \cos^2(\theta) = 1</math> and use it to find <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> given <math>\sin(\theta)</math>, <math>\cos(\theta)</math>, or <math>\tan(\theta)</math> and the quadrant of the angle.</p> <p><b>F.TF.C.9.</b> (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p>	
<p><b>21<sup>st</sup> Century Themes</b> Global Awareness Health Literacy</p>	<p><b>21<sup>st</sup> Century Skills</b> Critical Thinking and Problem Solving Communication and Collaboration</p>	
<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>● Glencoe Advanced Mathematical Concepts – Precalculus with Applications</li> <li>● Blitzer PreCalculus Text</li> <li>● Graphing Calculators</li> <li>● Document Camera</li> </ul>	
<p><b>Notes</b></p>		

<b>Grade: 11 - 12</b> <b>Subject: CP</b> <b>Pre-Calculus</b>	<b>Unit 6: Exponential and Logarithmic Functions</b>
<b>Big Idea/Rationale</b>	<p>Big Idea: Transcendental Functions</p> <p>Rationale: Transcendental functions go beyond their algebraic cousins and can be used to model real world phenomenon such as: population growth, radioactive decay, the spread of diseases, the Richter scale of earthquake intensity, the pH scale and the decibel measurement of sound.</p>
<b>Enduring Understanding (Mastery Objective)</b>	<ul style="list-style-type: none"> <li>● Exponents and logarithms are inverses of each other and their graphs are reflections of each other.</li> <li>● Exponential and logarithmic functions can be used to model behavior which will enable us to predict the future and rediscover the past.</li> <li>● Exponential, logarithmic and logistic functions are transcendental functions.</li> <li>● Transcendental Functions have wide spread applications.</li> </ul>
<b>Essential Questions (Instructional Objective)</b>	<ul style="list-style-type: none"> <li>● How can an exponential function represent a real-world scenario?</li> <li>● What is a logarithm?</li> <li>● How can the properties of logarithms be used to solve equations?</li> <li>● Why does simplifying or expanding a logarithmic expression help us solve problems?</li> <li>● Why is the number <math>e</math> important?</li> </ul>
<b>Content (Subject Matter)</b>	<p><i>Students will know...</i></p> <p><b>Key Terms</b> – scientific notation, properties of exponents, rational exponents, irrational exponents, power functions, exponential functions, exponential growth or decay, compound interest, the number <math>e</math>, continuously compounded interest, logarithmic function, logarithm, common logarithm, change of base formula, antilogarithm, natural logarithm, doubling time, nonlinear regression, linearization of data</p> <p><i>Students will be able to...</i></p> <ul style="list-style-type: none"> <li>● Simplify and evaluate expressions containing rational and irrational exponents</li> <li>● Use and graph exponential functions and inequalities</li> <li>● Evaluate expressions and graph and solve equations involving logarithms</li> <li>● Model real world situations and solve problems using common and natural logarithms</li> </ul>
<b>NJSLS Standards</b>	<p><b>N.RN.A.1.</b> Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. <i>For</i></p>



example, we define  $5^{1/3}$  to be the cube root of 5 because we want  $(5^{1/3})^3 = 5^{(1/3)3}$  to hold, so  $(5^{1/3})^3$  must equal 5.

**N.RN.A.2.** Rewrite expressions involving radicals and rational exponents using the properties of exponents.

**A.SSE.B.3.** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★

c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression  $1.15^t$  can be rewritten as  $(1.15^{1/12})^{12t} \approx 1.012^{12t}$  to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

**F.IF.C.7.** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

**F.IF.C.8.** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as  $y = (1.02)^t$ ,  $y = (0.97)^t$ ,  $y = (1.01)^{12t}$ ,  $y = (1.2)^{t/10}$ , and classify them as representing exponential growth or decay.

**F.BF.A.1.** Write a function that describes a relationship between two quantities.★

b. (+) Compose functions. *For example, if  $T(y)$  is the temperature in the atmosphere as a function of height, and  $h(t)$  is the height of a weather balloon as a function of time, then  $T(h(t))$  is the temperature at the location of the weather balloon as a function of time.*

**F.BF.B.5.** (+) Use the inverse relationship between exponents and logarithms to solve problems involving exponents and logarithms.

**F.LE.A.4.** Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to  $ab^{ct} = d$  where  $a$ ,  $c$ , and  $d$  are numbers and the base  $b$  is 2, 10, or  $e$ ; evaluate the logarithm using technology.

<p><b>21<sup>st</sup> Century Themes</b>          Global Awareness          Financial and Economic Literacy          Health Literacy</p>	<p><b>21<sup>st</sup> Century Skills</b>          Critical Thinking and Problem Solving          Communication and Collaboration</p>
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<p><b>Materials and Resources</b></p>	<ul style="list-style-type: none"> <li>● Glencoe Advanced Mathematical Concepts – Precalculus with Applications</li> <li>● Graphing Calculators</li> <li>● Document Camera</li> </ul>
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