ALGEBRA II ACTIVITY 14: ASYMPTOTES AND ZEROS OF RATIONAL FUNCTIONS

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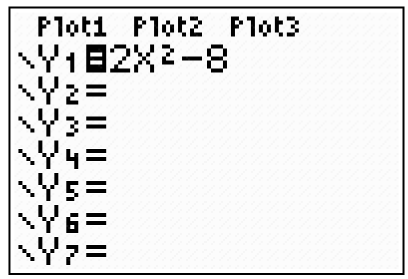
ACTIVITY OVERVIEW:
In this activity we will

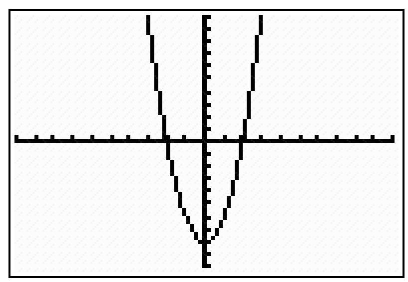
- Graph polynomial functions and examine their zeros and *y*-intercepts
- Analyze how the zeros and y-intercepts of the numerator and denominator affect the graph of a rational function

A rational function is the quotient of two polynomial functions where the polynomial function in the denominator is of degree 1 or higher. To understand the behavior of rational functions better, let's examine the polynomial functions that make them up.

Press [Y=] and enter the polynomial function shown. Later this function will become the *numerator* of a rational function.

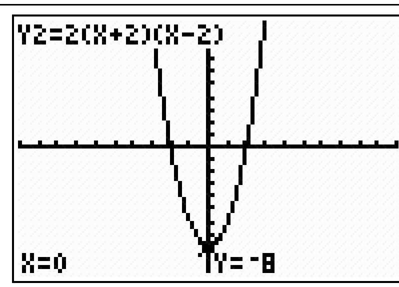


Press [GRAPH]. Examine the graph. Where are the zeros? What is the *y*-intercept?



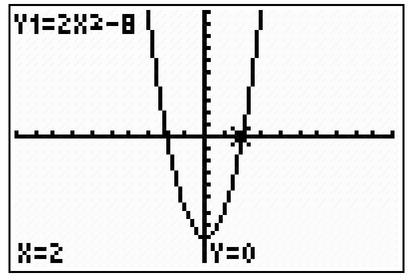
Use the trace feature to examine the graph closer. Find the *y*-intercept. Press [TRACE]and type in 0 (for *x*=0) [ENTER]. What indication did the equation give of what the *y*-intercept would be? Record this value.

Y-intercept for *numerator* is *y*= _____________

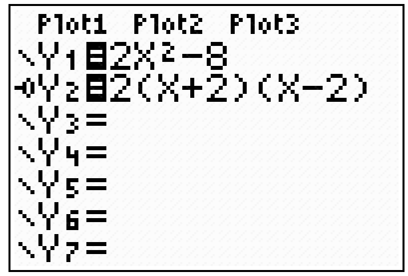


Continue to trace to find the zeros. Record.

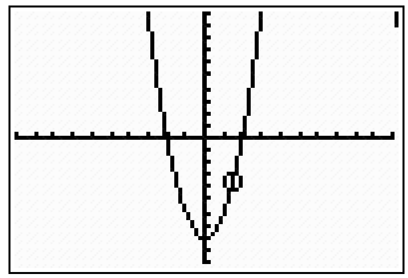
The *numerator* is zero at *x*= _______________



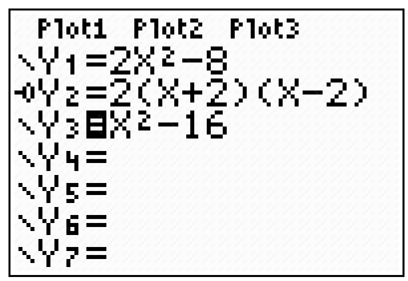
Press [Y=]. In **Y2** enter the factored form for $y=2x^2-8$. Left arrow to the left of **Y2** and press [ENTER] multiple times to change the format of the graph to be a circle that leaves a trail.



Press [GRAPH]. If the circle traces over the original graph of **Y1** then you know that you have the correct factored form in **Y2**. How does the factored form relate to the zeros of the function?

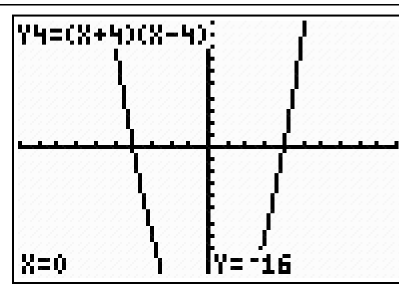


Press [Y=]. Move to the equals sign for **Y1** and press [ENTER] to turn the equation off. Do the same for **Y2**. Enter the function $y=x^2-16$ into **Y3**. Later this will become the *denominator* of our rational function.



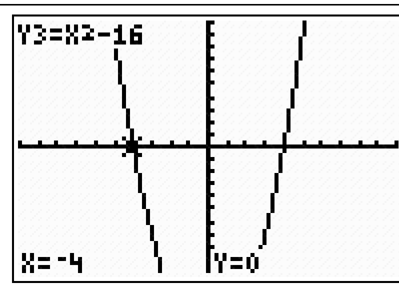
Press [GRAPH]. Press [TRACE] and type in 0 (for *x*=0) [ENTER]. What indication did the equation give of what the *y*-intercept would be? Record this value.

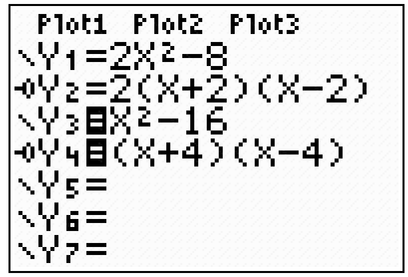
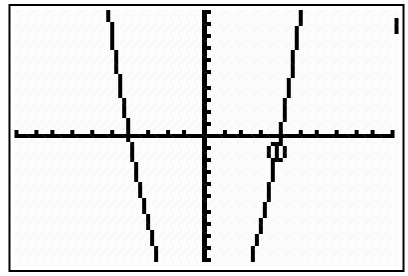
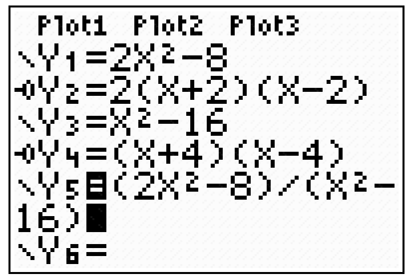
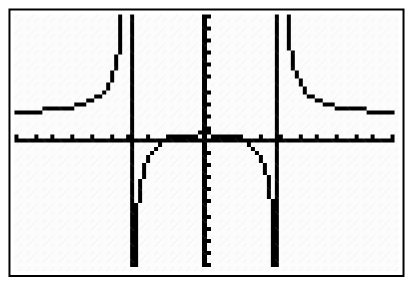
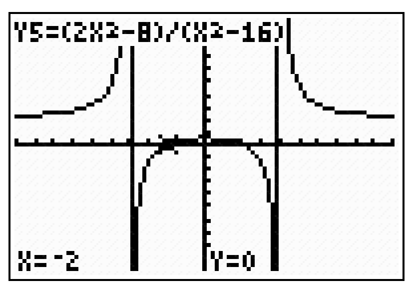
Y-intercept for *denominator* is *y*= _____________



Continue to trace to find the zeros. Record.

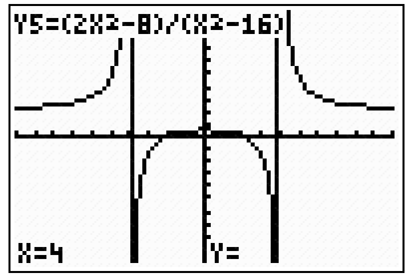
The *denominator* is zero at *x*= _______________



Press [Y=]. In **Y4** enter the factored form for $y=x^2-16$. Left arrow to the left of **Y4** and press [ENTER] multiple times to change the format of the graph to be a circle that leaves a trail.	
Press [GRAPH]. If the circle traces over the original graph of **Y3** then you know that you have the correct factored form in **Y4**. How does the factored form relate to the zeros of the function?	
Press [Y=]. Turn the equations **Y3** and **Y4** off. Enter the rational function $y=(2x^2-8)/(x^2-16)$ into **Y5**.	
Press [GRAPH]. On cursory inspection, where do you see interesting things happening on this graph? Is this a graph of a function?	
Press [TRACE]. Examine the behavior of the graph at some of the interesting values of *x*. Where do zeros of this function occur? The zeros of this function occur at the same locations as the zeros of the *numerator*. Why is this true?	

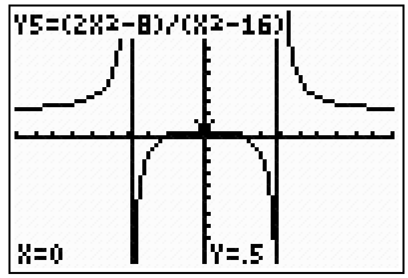
There appear to be some vertical lines on the graph. Where do these appear? While still in trace mode, type in 4 for x=4. What is the y-value at this point? What about when x=-4?

The things that appear to be vertical lines are not…they are *asymptotes* of the function. They occur because y is not defined for these values of x. Recall that 4 and -4 were the zeros of the *denominator*. What happens when the denominator of a fraction is zero?

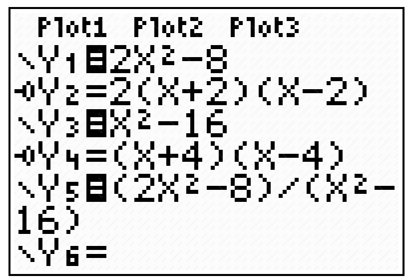


Lastly, look for the *y*-intercept. What is the value of *y* when *x* is 0?

Recall that the *y*-intercept of the *numerator* was -8 and the *y*-intercept of the *denominator* was -16. What is the quotient of these two values?



Press [Y=]. Turn the equations **Y1** and **Y3** back on.



Press [GRAPH]. If you look closely you can see that the zeros of the *numerator*'s parabola intersect the zeros of the rational function and that the zeros of the *denominator*'s parabola appear to cross the asymptotes at x=4 and -4. [Note: Asymptotes are normally invisible.]

Try this investigation with another set of polynomial functions.