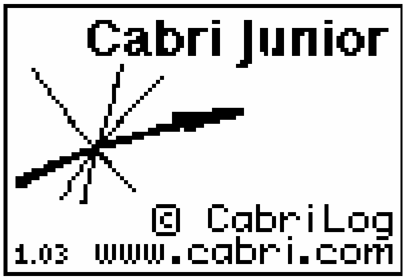
Activity 23 – President Garfield's Proof of the Pythagorean Theorem

First, turn on your TI-84 Plus and press the APPS key. Arrow down until you see Cabri Jr and press ENTER. You should now see this introduction screen.

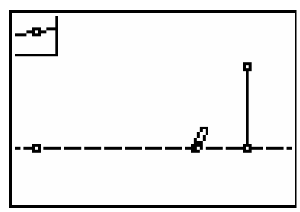


To begin the program, press any key. If a drawing comes up on the screen, press the Y= key (note the F1 above and to the right of the key – this program uses F1, F2, F3, F4, F5 names instead of the regular key names) and arrow down to NEW. It will ask you if you would like to save the changes. Press the 2nd key and then enter to not save the changes.

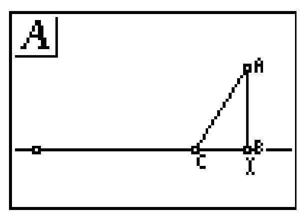
We are now ready to begin.

The following development of the relationships in a right angled triangle and the proof of the Pythagorean Theorem that follows are attributed to President James A. Garfield in 1876. First, let's explore his construction.

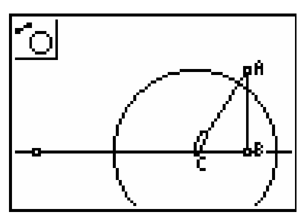
Construct a vertical line segment and a line perpendicular to the line segment. Use the "Point On" feature to add a point on the line.



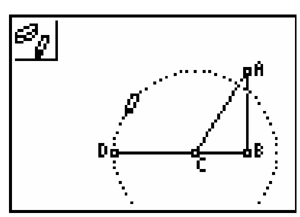
Label the new point C to create a right angled triangle ABC.



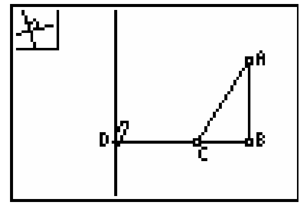
Use the compass tool to construct a circle with center at point C and radius AB.



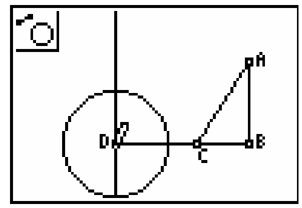
Find the point of intersection of the circle and the line. Label this point as D. We do not need the second point of intersection to the right of B. Hide the line, the circle and the second point of intersection.



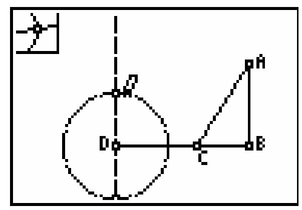
Construct a perpendicular to the line through point D.



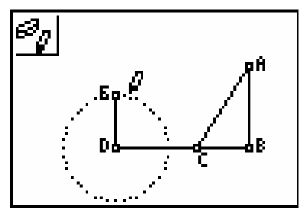
Use the compass tool again to construct a circle with center D and radius BC.



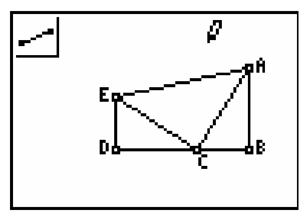
Construct the point(s) of intersection of this circle with the vertical line.



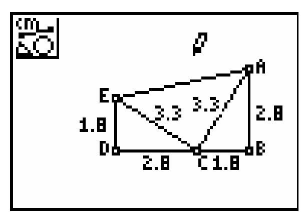
Label the point of intersection above D as E. Hide the vertical line the circle and the point of interseciton below D.



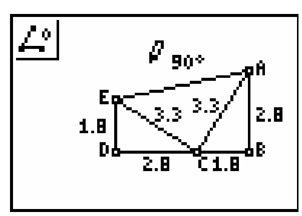
Construct a line segment connecting E to C. Due to the SAS congruence postulate, we now have two congruent triangles ABC and CBE. Thus, EC is congruent to AC. Construct a line segment connecting A to E.



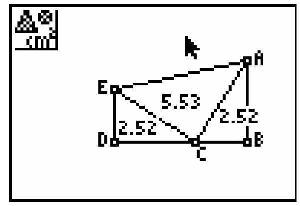
To confirm this result, measure the sides AB, BC, CD, DE, EC and AC.



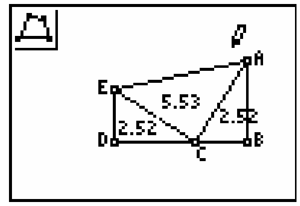
Measure angle ECA. It appears to be 90^o. Can you prove that this angle must be right angled?



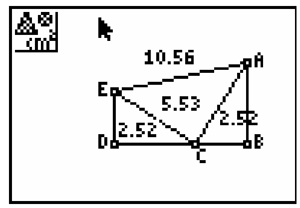
Construct triangles ABC, CDE and ECA. Measure the area of each triangle.



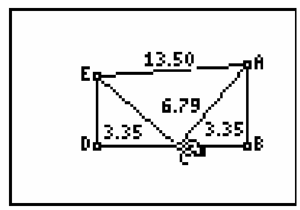
Use the Quadrilateral tool (Quad. in F2) to construct the figure passing through ABDE. Can you prove that this is a trapezoid? Which sides are the parallel bases of the trapezoid? Which side is the height?



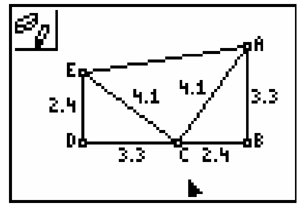
Measure the area of the trapezoid ABCE. Although it is not shown, add the areas of the three triangles ABC, CDE and ECA. Can you show that this sum must be equal to the area of the trapezoid?



Drag any of the points A, B or C to alter the construction. Do all of your properties hold?
ie: ABDE = ABC + CDE + ECA



In order to complete the proof, hide the areas and show the lengths of the sides again. Let's label them in the following manner:
Let AB = DC = x.
Let BC = ED = y and
Let EC = AC = z. We are trying to show that $x^2 + y^2 = z^2$.



In addition, let's find the area of the figures in the diagram in terms of x, y and z.
ABDE is a trapezoid with height DB = x + y and parallel sides ED = y and AB = x.
Thus, the area of ABDE = 0.5(x + y)(x + y)

ABC and CDE each have area 0.5(xy) and ECA has area 0.5(z*z) or $0.5(z^2)$

Now, use the fact that ABDE = ABC + CDE + ECA and substitute to get:

$0.5(x + y)(x + y) = 0.5(xy) + 0.5(xy) + 0.5(z^2)$ multiply each term by 0.5 to get:

$(x + y)(x + y) = (xy) + (xy) + (z^2)$ expand to get:

$x^2 + 2xy + y^2 = 2xy + z^2$ subtract 2xy from each side to get:

$x^2 + y^2 = z^2$

President Garfield lived in an era when it was consider appropriate for leaders to have a classical education, which included being well versed in mathematics.