



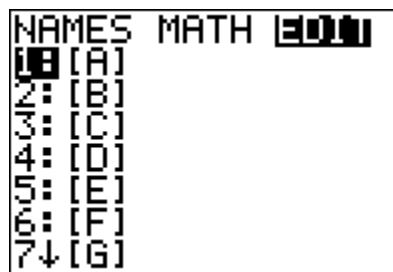
# ALGEBRA II ACTIVITY 10: SOLVING SYSTEMS USING MATRICES—THE LONG WAY AND THE SHORT WAY

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### ACTIVITY OVERVIEW:

In this activity we will

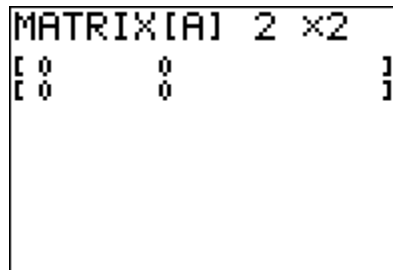
- Enter coefficients and constants from a system of equations into matrices
- Use row operations to solve the system using the row reduction method
- Use an inverse matrix to solve the system quickly



Consider the system of equations:

$$2x + y = 5$$
$$5x + 3y = 13$$

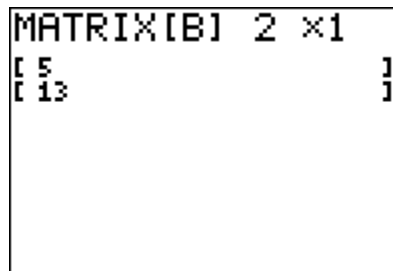
Since the equations are in standard form the coefficients can be entered into a matrix. Press  $\text{2nd}[x^{-1}]$  to access the **MATRIX** menu. Right arrow to **EDIT** and select **2: [A]**. Define Matrix A as a 2 row by 2 column matrix by typing over the dimensions in the top line. Press  $\text{ENTER}$  to see this screen.



Type in the coefficients of x in the first column and y in the second column as shown.



Next enter the constants into Matrix B. Press  $\text{2nd}[x^{-1}]$  to access the **MATRIX** menu. Right arrow to **EDIT** and select **2: [B]**. Enter the dimensions and the constants as shown.



Matrices A and B will be used later in the activity. To use the row reduction method, augment matrices A and B into Matrix C. Press  $\text{2nd}[\text{MODE}]$  to return to the home screen. Press  $\text{2nd}[\text{x}^{-1}]$  to access the **MATRIX** menu. Right arrow to **MATH** and select **7: augment(**.

```
NAMES [MATH] EDIT
1:det(
2:T
3:dim(
4:Fill(
5:identity(
6:randM(
7:augment(
```

This will paste the command **augment(** onto the home screen. Press  $\text{2nd}[\text{x}^{-1}]$  to access the **MATRIX** menu. Select **1:[A]**. Press  $\text{2nd}[\text{x}^{-1}]$  to access the **MATRIX** menu. Select **2:[B]**. Press  $\text{ENTER}$ .

```
augment([A],[B])
[[2 1 5 ]
 [5 3 13]]
```

Store the result in Matrix C. Press  $\text{STO} \rightarrow \text{2nd}[\text{x}^{-1}]$  and select **3:[C]**. Press  $\text{ENTER}$ .

```
augment([A],[B])
[[2 1 5 ]
 [5 3 13]]
Ans→[C]
[[2 1 5 ]
 [5 3 13]]
```

The next step to solve the system would be to eliminate  $x$  by multiplying the first equation by  $-2.5$  and adding it to the second equation. This can be done using row operations.

Press  $\text{2nd}[\text{x}^{-1}]$  to access the **MATRIX** menu. Right arrow to **MATH** and select **F: \*row+(**.

(This command multiplies a row and adds it to another row.)

```
NAMES [MATH] EDIT
0:cumSum(
A:ref(
B:rref(
C:rowSwap(
D:row+(
E:*row(
F:*row+(
```

Press  $(-)[2][.][5][,][\text{2nd}[\text{x}^{-1}][3][,][1][,][2][)]$ . This enters the command shown, which means: "multiply by  $-2.5$  Matrix C's first row, and add the result in the second row." Press  $\text{ENTER}$ . This will NOT replace Matrix C.

```
*row+(-2.5,[C],1
,2)
[[2 1 5 ]
 [0 .5 .5]]
```

Store the result in Matrix D. Press  $\text{STO} \rightarrow \text{2nd}[\text{x}^{-1}]$  and select **4:[D]**. Press  $\text{ENTER}$ .

```
*row+(-2.5,[C],1
,2)
[[2 1 5 ]
 [0 .5 .5]]
Ans→[D]
[[2 1 5 ]
 [0 .5 .5]]
```

The goal is to get the coefficient of  $y$  to be 1. To do this requires doubling the second row. Press  $\boxed{2\text{nd}}\boxed{\times^{-1}}$  to access the **MATRIX** menu. Right arrow to **MATH** and select **E: \*row(**.

(This command multiplies a row.)

```
NAMES [MATH] EDIT
0: cumSum(
A: ref(
B: rref(
C: rowSwap(
D: row+(
E: *row(
F: *row+(
```

Press  $\boxed{2\text{nd}}\boxed{,}\boxed{2\text{nd}}\boxed{\times^{-1}}\boxed{4}\boxed{,}\boxed{2}\boxed{)}$ . This enters the command as shown at the top of the screen shown. The command means: “multiply by 2 Matrix D’s second row.” Press  $\boxed{\text{ENTER}}$ . This does NOT replace Matrix D.

Store the result in Matrix D (replace it) by pressing  $\boxed{\text{STO}}\boxed{\blacktriangleright}\boxed{2\text{nd}}\boxed{\times^{-1}}\boxed{4}\boxed{\text{ENTER}}$ .

```
      [0  .5  .5]
*row(2, [D], 2)
      [[2  1  5]
       [0  1  1]]
Ans→ [D]
      [[2  1  5]
       [0  1  1]]
```

The next step to solve the system would be to eliminate  $y$  by multiplying the second equation by  $-1$  and adding it to the first equation. This can also be done using row operations.

Press  $\boxed{2\text{nd}}\boxed{\times^{-1}}$  to access the **MATRIX** menu. Right arrow to **MATH** and select **F: \*row+(**.

Enter the command as shown, which means: “multiply by  $-1$  Matrix D’s second row and add it to the first row.”

Store the result in Matrix D.

```
*row+( -1, [D], 2, 1
)
      [[2  0  4]
       [0  1  1]]
Ans→ [D]
      [[2  0  4]
       [0  1  1]]
```

The goal here is to get the coefficient of  $x$  to be 1. To do this requires halving the first row. Press  $\boxed{2\text{nd}}\boxed{\times^{-1}}$  to access the **MATRIX** menu. Right arrow to **MATH** and select **E: \*row(**.

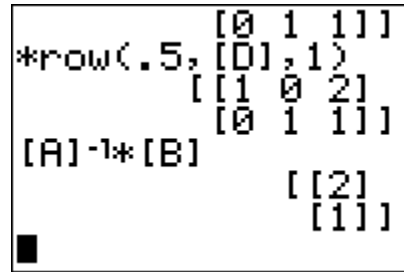
Enter the command as shown, which means: “multiply by  $0.5$  Matrix D’s first row.”

The resulting matrix is in reduced row-echelon form. The last column indicates the solution to the system is the coordinate pair  $(2,1)$ .

```
*row(.5, [D], 1)
      [[1  0  2]
       [0  1  1]]
```

Once students understand the row reduction method, they may wish to know the quick method for using matrices to solve systems. Recall that Matrix A contains the coefficients of x and y and Matrix B contains the constants.

Press  $\boxed{2\text{nd}}\boxed{x^{-1}}\boxed{1}$  to bring Matrix A to the home screen. Press  $\boxed{x^{-1}}$  to instruct the calculator to take its inverse. Multiply the Matrix B by pressing  $\boxed{\times}\boxed{2\text{nd}}\boxed{x^{-1}}\boxed{2}\boxed{\text{ENTER}}$ . The resulting matrix contains the solution.



The image shows a TI-84 Plus calculator screen with the following text and matrices displayed:

```
*row(.5, [0 1 1])  
          [0] 1)  
          [[1 0 2]  
          [0 1 1])  
[A]-1*[B]          [[2]  
                   [1])
```