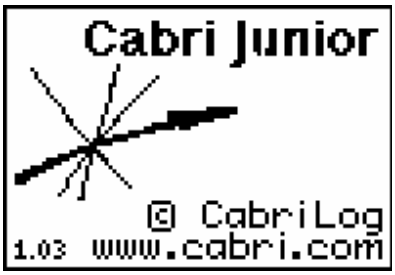
Activity 11 Exterior Angle and Side-Angle Relationships

First, turn on your TI-84 Plus and press the APPS key. Arrow down until you see Cabri Jr and press [ENTER]. You should now see this introduction screen.



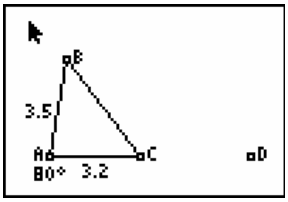
To begin the program, press any key. If a drawing comes up on the screen, press the [Y=] key (note the F1 above and to the right of the key – this program uses F1, F2, F3, F4, F5 names instead of the regular key names) and arrow down to NEW. It will ask you if you would like to save the changes. Press the [2nd] key and then enter to not save the changes.

We are now ready to begin.

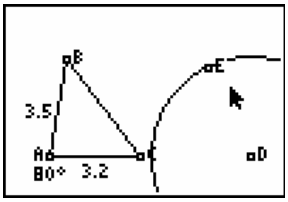
We have looked at triangle congruence postulates earlier. Now, we turn our attention to the inequality relationships that arise when some of the congruence conditions are in place but others are not.

SAS Inequality Theorem: Two sides of a triangle are congruent to two sides of another triangle. If the included angle in the first is greater than the included angle of the second triangle, then the third side of the first triangle is longer than the third side of the second triangle.

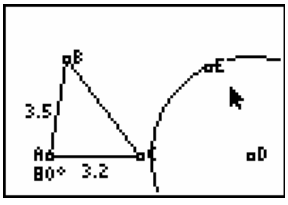
Start by constructing any triangle ABC and a fourth point D. To avoid any incorrect assumptions, be sure that triangle ABC is a scalene triangle. Measure sides AB, AC and angle A.



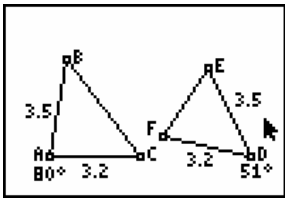
Use the compass tool to construct a circle with center D and radius AB. Any point E on this new circle will allow us to construct a line segment DE that has the same length as AB. Hide the circle.



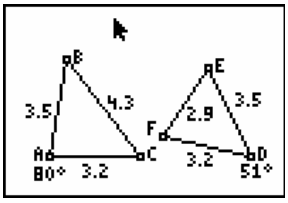
Use the compass tool to construct a circle with center D and radius AC. Any point F on this new circle will allow us to construct a line segment DF that has the same length as AC. Hide the second circle as well.



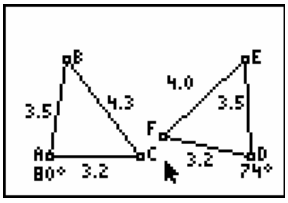
Construct line segments DE, DF and EF or construct a triangle with vertices D, E and F. Measure sides DE and DF. They should have the same lengths as AB and AC respectively. Measure angle EDF. You may need to drag either point E or point F to ensure that angle BAC > angle EDF.



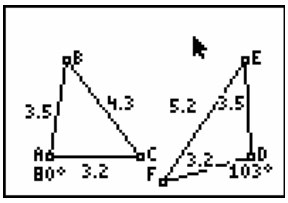
Measure the third side in each triangle, that is side BC and side EF. Verify that the property stated in the theorem holds for this example. Since angle BAC > angle EDF, then side BC > side EF.



Drag point F so that angle EDF increases in size but remains smaller than angle BAC. What happens to the relationship between sides BC and EF as angle EDF gets closer to angle BAC?

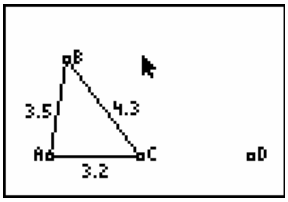


If angle EDF = angle BAC, what property holds? What happens to the relationships when angle EDF > angle BAC? How does this affect the statement of the theorem?

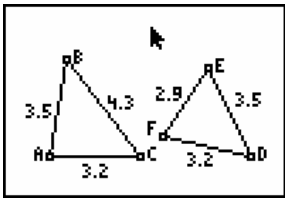


SSS Inequality Theorem: Two sides of a triangle are congruent to two sides of another triangle. If the third side in the first triangle is greater than the third side of the second triangle, then the angle between the congruent sides of the first triangle is greater than the angle between the congruent sides of the second triangle.

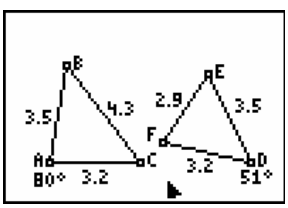
To demonstrate this property, construct a scalene triangle ΔABC and a fourth point D. Measure the sides of ΔABC.



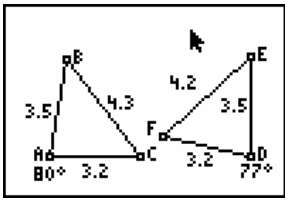
Using the compass tool as before, construct points E and F so that ED = AB and DF = AC. Complete the triangle ΔDEF. Measure the sides of ΔDEF. You may have to drag either point E or point F so that BC > EF.



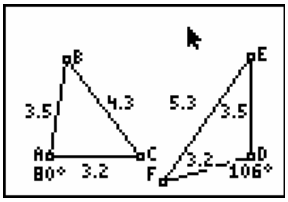
Measure angle BAC and angle EDF. In the diagram, the statement of the theorem holds since angle BAC > angle EDF.



Drag either point E or point F so that the length of EF approaches the length of BC. What happens to the relationship between the corresponding angles?



What happens when EF = BC? We have seen this before. Finally, what happens to the relationship between the angles if you continue to drag either point E or point F so that EF > BC?



Is there a similarity between the two theorems mentioned in this activity? Could we consider the congruence of triangles just a spin off of the inequality relationships?