Addition and Multiplication with Volume and Area

In Module 6, students begin by reasoning about and working with three-dimensional shapes. They explore cubic units and move toward calculations of volumes of rectangular prisms. Students also extend their two-dimensional work with area to figures with fractional side lengths. This module bridges the Grade 4 work on area with the Grade 6 work on volume and area to come.

What Came Before this Module: Students learned to multiply fractions and decimal fractions and began work on fraction division, working from concrete to abstract representations.

What Comes After this Module: In Module 6, students begin to explore the coordinate plane, working from the familiar number line toward plotting points and creating lines and patterns.

New Terms in this Module:
- Base: one face of a three-dimensional solid—often thought of as the surface upon which the solid rests
- Bisect: divide into two equal parts
- Cubic units: cubes of the same size used for measuring
- Height: adjacent layers of the base that form a rectangular prism
- Hierarchy: series of ordered groupings of shapes
- Unit cube: cube whose sides all measure 1 unit
- Volume of a solid: measurement of space or capacity

Key Common Core Standards:
- Apply and extend previous understanding of multiplication and division to multiply and divide fractions.
  - Multiply a fraction or whole number by a fraction.
  - Solve real world problems involving multiplication of fractions and mixed numbers.
- Geometric measurement: understand concepts of volume and relate volume to multiplication and addition.
  - Recognize volume as an attribute of solid figures and understand concepts of volume measurement.
  - Measure volumes by counting unit cubes of various units.
  - Relate volume to the operations of multiplication and addition.
- Classify two-dimensional figures into categories based on their properties.
  - Understand that attributes belonging to a category of figures also belong to all subcategories of that category.

How You Can Help at Home:
- Begin to discuss and notice the volume of various household containers—this is also a good opportunity to talk about what units are often used to measure volume.
- Keep practicing those multiplication and division facts, especially as problems become more complex.

An area calculation for $3\frac{1}{2} \times 1\frac{1}{4}$

Two orientations of 12 unit cubes

Unit Cubes
Earlier in Grade 5, we moved beyond using the area model for multiplication of whole numbers and begin to use this powerful model to illustrate mathematical operations on fractions. Now, we move a step further and use the area model in various real world problems, e.g., finding the area of a wall minus the space for two windows, or finding the area of a mat surrounding a picture in a frame.

The numbers we use in our area models now are often mixed whole numbers and fractions, giving students a chance to demonstrate their understanding in diagrams in which they show the multiplication of both the whole number and fractional parts of the problem.

**Sample Volume Problem from Module 5:**
(Example taken from Module 5, Lesson 18)

How many 2-inch cubes are needed to build a rectangular prism that measures 10 inches by 6 inches by 14 inches?

`3 \times 5 \times 7 = 105`

Note that in the area problem above, the numbers are first decomposed and drawn as whole number and fractional parts, and then they are multiplied: $1 \times 3$, $1/3 \times 3$, $1 \times 3/4$, $1/3 \times 3/4$. Each of these products is then added together to find the total area of the rectangle.

**A Story of Units** has several key mathematical “models” that will be used throughout a student’s elementary years.

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Note that the student here shows two ways to solve the problem!