The Locker Problem

The Scenario:

It is the first day of middle school. There are 1000 students and 1000 lockers. All of the lockers are closed. Student #1 walks down the hallway and opens all of the lockers and then leaves. Student #2 walks down the hallway afterwards and closes all the lockers that are multiples of 2 (2, 4, 6, 8, 10, ..., 998, 1000). Then Student #3 walks down the hallway and CHANGES THE STATE (if the lockers are open, closes them, if they are closed, she opens them) of all of the multiples of 3 (3, 6, 9, 12, ..., 996, 999). Student #4 then walks down the hall and changes the state of all the multiples of 4. This process continues until ALL 1000 STUDENTS have walked down the hallway and changed the state of their multiples.

The Question:

At the end, how many lockers are open, which ones, and why?

The Donut Problem

You are buying a dozen donuts from a donut shop. There are three types of donuts: glazed, jelly-filled and chocolate. How many different flavor combinations of a dozen donuts could you get? (Note: You do NOT have to get one of each flavor, so you could just get a dozen glazed for example.)