11.3: Arithmetic and Geometric Sequences and Series Word Problems

Formulas

Arithmetic Sequence $a_n = a_1 + (n-1)d$

Arithmetic Series: $S_n = \frac{n}{2}(2a_1 + (n-1)d)$ or $S_n = \frac{n(a_1+a_n)}{2}$

Geometric Sequence $a_n = a_1 r^{n-1}$

Finite Geometric Series $S_n = \frac{a_1(1-r^n)}{1-r}$

a_1 is the FIRST TERM
a_n is the TERM you are looking for
n is the NUMBER of the TERM you are looking for
d is the COMMON DIFFERENCE
r is the COMMON RATIO
S_n is the SUM of all the terms in the sequence

Examples:

1) After knee surgery, your trainer tells you to return to your jogging program slowly. He suggests jogging for 12 minutes each day for the first week. Each week thereafter, he suggests that you increase that time by 6 minutes per day. How many weeks will it be before you are up to jogging 60 minutes per day?

$a_1 = 12$
$d = 6$
Find n when $a_n = 60$

$a_n = a_1 + (n-1)d$
$60 = 12 + (n-1)6$
$60 = 12 + 6n - 6$
$60 = 6 + 6n$
$54 = 6n$
$9 = n$
9 weeks.

2) A theater has 60 seats in the first row, 68 seats in the second row, 76 seats in the third row, and so on in the same increasing pattern. If the theater has 20 rows of seats, how many seats are in the 20th row of the theater?

$a_1 = 60$
$d = 8$
Find a_{20}

$a_n = a_1 + (n-1)d$
$a_{20} = 60 + (20-1)8$
$a_{20} = 60 + (19)8$
$a_{20} = 60 + 152$
$a_{20} = 212$

3) The sum of the first 6 terms of a geometric sequence is 7812. The common ratio is 5. What is the **second** term of the sequence?

$S_6 = 7812$
$r = 5$
Find a_2

$S_n = \frac{a_1(1-r^n)}{1-r}$
$7812 = \frac{a_1(1-5^6)}{1-5}$

$7812 = \frac{-15624a_1}{-4}$
$7812 = 3906a_1$
$2 = a_1$
$a_2 = a_1 * r$
$a_2 = 2 * 5$
$a_2 = 10$

ASSIGNMENT: Label 11.3 Word Problems

1) A brick wall has 60 bricks in the bottom row but each row has 3 fewer bricks than the previous one. How many bricks are in the 12th row?

2) Which of the following represents as arithmetic sequence? Circle the correct answer.
      A) All of the multiples of 5 between 10 and 100
      B) All of the even integers between 1 and 100
      C) All of the odd integers between 1 and 100
      D) All of the answers are correct

3) The second term of a Geometric sequence is 25 and the third term is 125. What is the first term in the series?

4) The first term of Geometric sequence is 5 the ratio between subsequent numbers is 2. What is the fourth term in the sequence?

5) A display of cans on a grocery shelf consists of 20 cans on the bottom, 18 cans in the next row, and so on in an arithmetic sequence, until the top row has 4 cans. How many cans, in total, are in the display?

6) Brian gets a starting wage of $15 and an annual raise of $1.50 per hour. What will Brian's hourly wage be during his tenth year? (Hint: How many years has he worked when he starts out earning $15?)

7) A pile of bricks has 97 bricks in the first row, 91 bricks in the second row, 85 bricks in the third row, and so on until there is only one brick in the top row. How many bricks are in the 15th row?