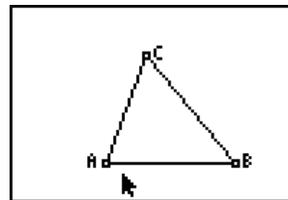Investigating the Triangle Inequality

One property of a triangle is that the sum of the lengths of any two sides must be greater than the length of the third side. We will investigate this in two ways.

First, start with a triangle. Use the Alph-Num tool to label the three vertices.

Find the lengths of the three sides of the triangle using the Measure D. & Length tool. Let the tool rest on each side until just the one side is active. This way, you'll be sure that you are measuring each side rather than the perimeter.
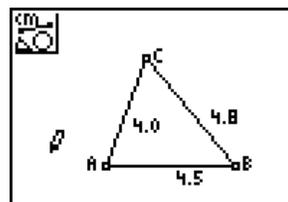
Use the Alph-Num tool to create labels for the sums of each pair of sides in the triangle.
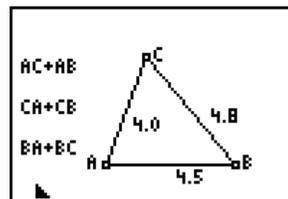
Find the sum of each pair of sides. Compare each sum to the length of the third side. Is it possible to drag one of the vertices so that the sum of two sides is greater than the third side?
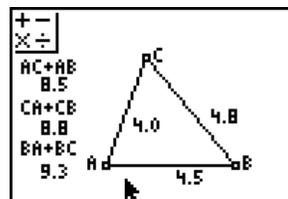
To show this property in a different manner, start with a line segment at the bottom of the screen. Label the endpoints P and S. Use the Point On tool to construct two points on the line segment. Label these points Q and R. Construct line segments PQ, QR and RS and find the lengths of these line segments.
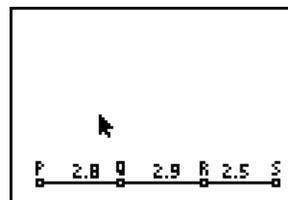
We will be using the three line segments PQ, QR and RS to construct a triangle with sides of the same lengths. To begin, construct a line segment anywhere on the screen. We will use this line segment as the base of the triangle. However, we need to copy the length of PQ onto this segment.
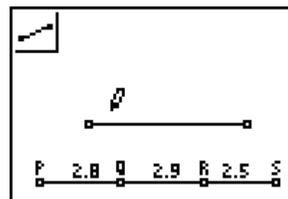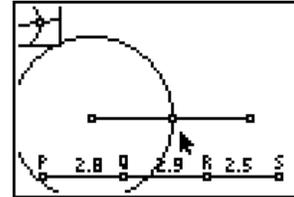
We will use the Compass tool to cut the line segment so that the result has the same length as PQ. Press ‾ and select Compass.
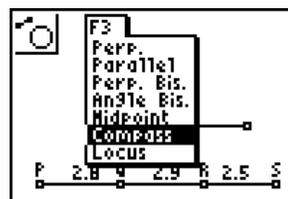
With the compass tool active, select the left endpoint of the new line segment and then, select line segment PQ. This will construct a circle with a radius equal to PQ.
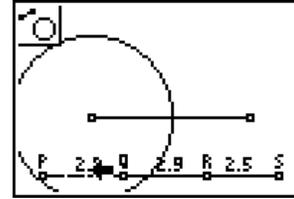
Press _ and select the Intersection option from the Point sub-menu. Using this tool, find the point of intersection of the new line segment and the circle.

Hide the circle, the line segment and the right endpoint of the line segment. Construct a new line segment and label it AB. Drag point Q and describe what happens to line segment AB.
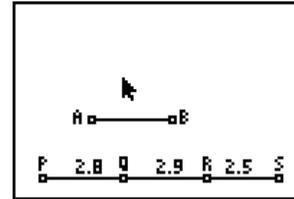
Use the compass tool again to construct two new circles. For the first circle, use point A as the center and line segment QR as the radius. For the second circle, use point B as the radius and line segment RS as the radius.
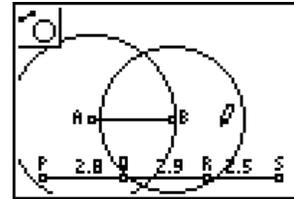
Find the point of intersection of the two circles. You may see two points of intersection. Hide the circles and the second point of intersection. Label this new point C.
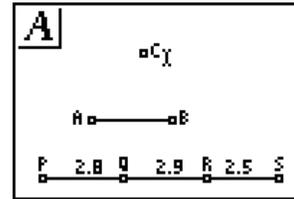
Construct line segments AC and BC to complete the triangle. You now have a triangle with lengths AB = PQ, AC = QR and BC = RS. If you drag either Q or R, the triangle ABC will change accordingly.
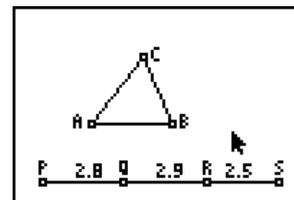
As long as the sum of the lengths of two of the sides is greater than the length of the third side, you will have a triangle. However, drag point Q to the left and point R to the right until PQ + RS < QR. What happens to the figure? Will this always happen? What happens when PQ + RS = QR?