

Advanced Algebra and Trigonometry Curriculum Maps

Unit 1: Review of Basic Algebra Concepts

Unit 2: Systems and Matrices

Unit 3: Operations with Polynomials of nth Degree ($n \geq 2$)

Unit 4: Rational and Radical Functions

Unit 5: Solving and Graphing Polynomials and Rational Equations and Inequalities

Unit 6: Inverse, Exponential, & Logarithmic Functions

Unit 7: Trigonometric Functions and Their Graphs

Unit 8: A Further Study of Trigonometry

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 1: Review of Basic Algebra Concepts
Big Idea/Rationale	Students will review basic Algebra concepts that will be necessary for further instruction in the Advanced Algebra and Trigonometry course. Students will be expected to apply concepts from this unit in future lessons and course work.
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Identify the sets in the real number system to which a number is an element of. • Solve linear and absolute value equations and inequalities, including use of formulas. • Define the domain and range of a function. • Use proper function notation.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • Can a number be an element of more than one set? • Are specific sets of the real number system contained within others? • What does the solution of a linear equation or linear inequality represent? • How are absolute value equations and inequalities related to linear equations and inequalities? • What are independent and dependent variables? • What do the domain and range of a relation represent? • What is the difference between a relation and a function?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Terms – Set, element, natural numbers, counting numbers, whole numbers, rational numbers, irrational numbers, real numbers, solution, solution set, equivalent equations, conditional equations, identities, contradictions, mathematical model, formula, factor, percent, inequality, equivalent inequalities, three-part inequalities, independent variable, dependent variables, relation, function, domain, range, Vertical Line Test, function notation, linear function, constant function</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Identify the common sets of numbers. • Decide whether a number is a solution of a linear equation. • Solve linear equations. • Identify conditional equations, contradictions, and identities. • Solve a formula for a specified variable. • Solve applied problems by using formulas. • Solve percent problems. • Translate from words to mathematical expressions. • Write equations from given information.

	<ul style="list-style-type: none"> • Solve applied problems including percent, investment and mixture problems. • Solve linear inequalities. • Solve applied problems using linear inequalities. • Solve absolute value equations and inequalities. • Distinguish between independent and dependent variables. • Define and identify relations and functions. • Find the domain and range. • Identify functions defined by graphs and equations. • Use function notation. • Graph linear and constant functions.
<p>NJSLS Standards</p>	<p>A.REI.A.1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.</p> <p>A.REI.B.3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p> <p>A.CED.A.2. Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.</p> <p>A.CED.A.4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm’s law $V = IR$ to highlight resistance R.</p> <p>F.IF.A.1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.</p> <p>F.IF.A.2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</p> <p>F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function</p>
<p>21st Century Themes</p>	<p>21st Century Skills Creativity and Innovation</p>

Global Awareness Financial and Economic Literacy	Critical Thinking and Problem Solving Communication and Collaboration
Materials and Resources	Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector
Notes	

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 2: Systems and Matrices
Big Idea/Rationale	<p>Systems of equations can be used to solve many real-life problems in which multiple constraints are used on the same variables. Matrix operations provide an alternate way of solving systems of equations. This method can prove more efficient especially with the aid of a graphing calculator. Matrix mathematics can be applied in computer graphics, science and other areas of mathematics such as graph theory, probability and statistics, and business math.</p>
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Solve systems of equations graphically and algebraically. • Define a matrix and use it to model data. • Use row operations to solve a system. • Find the inverse of a matrix and use it to solve a system on your graphing calculator.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • What does the number of solutions (none, one or infinite) of a system of linear equations represent? • What are the advantages and disadvantages of solving a system of linear equations graphically versus algebraically? • How can systems of equations be used to represent situations and solve problems? • What are the advantages and disadvantages of organizing data in a matrix? • How are augmented matrices put in row echelon form? • How do we find the inverse of a matrix and when does a matrix not have an inverse defined? • How can we use the graphing calculator to help us solve systems using a matrix?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Key Terms - system of equations, consistent system, inconsistent system, independent system, dependent system, ordered triple, matrix, element of a matrix, row, column, dimensions of a matrix, square matrix, augmented matrix, row operations, row echelon form, inverse matrix</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Solve a system of linear equations with two and three variables. • Write the augmented matrix of a system. • Use row operations to solve a system. • Use a graphing calculator to find the inverse of a matrix. • Solve a system using inverse matrices.

<p>NJSLS Standards</p>	<p>A-CED.A.3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. <i>For example, represent inequalities describing nutritional and cost constraints on combinations of different foods</i></p> <p>A-REI.C.6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.</p> <p>A-REI.C.8. (+) Represent a system of linear equations as a single matrix equation in a vector variable.</p> <p>A-REI.C.9. (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3×3 or greater).</p> <p>N-VM.C.6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.</p>	
<p>21st Century Themes Financial and Economic Literacy</p>	<p>21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration</p>	
<p>Materials and Resources</p>	<p>Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector</p>	
<p>Notes</p>		

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 3: Operations with Polynomials of nth Degree ($n \geq 2$)
Big Idea/Rationale	Students will apply their knowledge of operations with numbers to operations with polynomials. They will extend this knowledge to understand how to solve real world application problems.
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Perform operations of addition, subtraction, and multiplication of polynomials. • Compose functions based on real world applications, such as retail. • Rewrite rational expressions in polynomial form using operation of division. • Use factoring as a key method in solving polynomial equations.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • How are operations on polynomials similar to operations on integers? • What does the composition of functions represent in a real world situation? • Why do we use function operation notation? • What are the different methods for factoring polynomials? • When can we apply factoring polynomials to problem solving applications?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Terms – term, numerical coefficient, algebraic expression, polynomial, descending powers, monomial, binomial, trinomial, degree of a term, degree of a polynomial, like terms, operations on functions, composite functions, composition, product of the sum and difference of two terms, square of a binomial, long division, synthetic division, factoring, factor by grouping, prime polynomial, difference of squares, perfect square trinomial, difference of cubes, sum of cubes, zero product property, standard form</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Find the degree of a polynomial. • Add and subtract polynomials. • Use a polynomial function to model data. • Find the composition of functions. • Multiply polynomial functions. • Find the product of the sum and difference of two terms. • Find the square of a binomial. • Divide polynomial functions. • Factor out the greatest common factor. • Factor by grouping.

	<ul style="list-style-type: none"> • Factor trinomials. • Factor by substitution. • Factor a difference of squares. • Factor a perfect square trinomial. • Factor the sum and difference of cubes. • Learn and use the zero product property. • Solve applied problems that require the zero product property.
NJSLS Standards	<p>A.SSE.A.1.a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>A.APR.A.1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</p> <p>F.BF.A.1.c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.</p> <p>A.APR.D.6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.</p> <p>A.SSE.A.2. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.</p> <p>A.SSE.B.3.a. Factor a quadratic expression to reveal the zeros of the function it defines.</p> <p>A.REI.B.4.b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b.</p>
21st Century Themes Global Awareness Financial and Economic Literacy Health Literacy	21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration
Materials and Resources	Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector
Notes	<ul style="list-style-type: none"> • Synthetic division from Section 12.1 added to Section 5.5. • Section 6.4 is a review of factoring.

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 4: Rational and Radical Functions
Big Idea/Rationale	<p>The focus of this unit is solving rational and radical functions. The skills learned in this chapter will be needed for success in future math classes such as Precalculus. Outside of school students can use these skills to make predictions involving time, distance, money, or speed.</p>
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Performing operations with rational expressions is a fundamental skill for solving rational equations and inequalities. • Operations with rational expressions follow the same rules as operations with fractions. • Radical equations and inequalities are found in many scientific formulas.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • What values make up the domain of a rational expression? • How is adding and subtracting rational expressions similar to adding and subtraction fractions? • How do we decide which method is most appropriate when solving rational equations? • How does the domain help you determine whether the solution to a radical or rational equation is extraneous?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Terms – rational expression, rational function, fundamental property of rational numbers, reciprocal, least common denominator, complex fraction, rational equation, domain of the variable in a rational equation, radicand, index, order, radical, principal root, square root function, cube root function, product rule for radicals, quotient rule for radicals, simplified radicals, Pythagorean Theorem, distance formula, rationalizing the denominator, conjugates, radical equation, power rule, imaginary unit i, complex number, real part, imaginary part, complex conjugate</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Write rational expressions in lowest terms. • Multiply rational expressions. • Find reciprocals of rational expressions. • Divide rational expressions. • Find a least common denominator. • Add and subtract rational expressions with the same and different denominators. • Simplify complex numbers by simplifying the numerator and denominator.

	<ul style="list-style-type: none"> • Simplify complex numbers by multiplying by a common denominator. • Simplify rational expressions with negative exponents. • Determine the domain of the variable in a rational equation. • Solve rational equations. • Find the value of an unknown variable in a formula. • Solve applications by using proportions. • Solve applications about distance, rate, and time. • Solve applications about work rates. • Find roots and principal roots of numbers. • Graph functions defined by radical expressions. • Find nth roots of nth powers. • Convert between radicals and rational exponents. • Use the rules for exponents with rational exponents. • Simplify radicals using the product rule and quotient rule. • Simplify products and quotients of radicals with different indexes. • Apply knowledge of simplifying radicals to the Pythagorean Theorem and distance formula. • Simplify radical expressions involving addition and subtraction. • Multiply radicals. • Rationalize denominators. • Write radical quotients in lowest terms. • Solve radical equations by using the power rule. • Solve radical equations by using the graphing calculator. • Use the power rule to solve a formula for a specified variable. • Simplify imaginary numbers. • Recognize complex numbers. • Add, subtract, multiply, and divide complex numbers. • Find powers of i.
<p>NJSLS Standards</p>	<p>A.APR.D.7. (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.</p> <p>A.REI.A.2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.</p> <p>A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</p> <p>A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>N.RN.A.1. Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational</p>

	<p>exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.</p> <p>N.RN.A.2. Extend the properties of exponents to rational exponents. Rewrite expressions involving radicals and rational exponents using the properties of exponents.</p> <p>F.IF.C.7.b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>N.CN.A.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.</p> <p>N.CN.A.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p>
<p>21st Century Themes Global Awareness Financial and Economic Literacy</p>	<p>21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration</p>
<p>Materials and Resources</p>	<p>Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector</p>
<p>Notes</p>	<ul style="list-style-type: none"> • Skip Section 7.6: Variation Functions • Graphs of Rational Functions will be covered in Section 12.4

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 5: Solving and Graphing Polynomials and Rational Equations and Inequalities
Big Idea/Rationale	Polynomial and rational function families are used extensively to model behavior in the social, behavioral and natural sciences. The theorems covered in this unit, including the fundamental theorem of algebra and the rational zeros theorem, give a fairly comprehensive view of the study of polynomial functions.
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Polynomial functions arise naturally in many applications. • Many complicated functions can be approximated by polynomial functions or their quotients, rational functions. • Algebraic methods can be used for finding real number and complex number solutions for polynomial and rational functions. • The graphical behavior of these functions is directly related to the function. • The characteristics of polynomial functions and their representations are useful in solving real-world problems. • Rational equations often result in extraneous solutions. • Methods of solving polynomial and rational equations can be extended to inequalities.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • What are the zeros of a polynomial function and how can we find them? • Explain how to use zeros and end behaviors to sketch a possible graph of a polynomial function. • How can you determine the multiplicity of real roots of a polynomial from its graph? • What is a complex number and how do you perform operations with complex numbers? • While solving quadratic equations by factoring, why is each factor set equal to zero? • What is the significance of the determinant in the Quadratic Formula? • Explain how to sketch the graph of a quadratic function given in vertex form. • Explain how to sketch the graph of a quadratic function given in standard form. • What are the limitations of synthetic division? • How are the Remainder and Factor Theorems related? • Why must all solutions of a rational equation be checked? • How do we transform and graph rational functions? • How do we solve polynomial inequalities?

**Content
(Subject Matter)**

Student will know.....

Terms – quadratic equation, standard form, completing the square, quadratic formula, discriminant, quadratic form, quadratic inequality, difference quotient, composite function, vertex, axis of symmetry, horizontal translation, vertical translation, symmetric, absolute value function, piecewise linear function, greatest integer function, step function, remainder theorem, polynomial function of degree n , factor theorem, rational zeros theorem, fundamental theorem of algebra, number of zeros theorem, multiplicity, turning points, conjugate zeros theorem, end behavior, dominating term, intermediate value theorem, boundness theorem, rational function, discontinuous graph, vertical asymptote, horizontal asymptote, oblique asymptote, “hole” or point of discontinuity.

Student will be able to.....

- Solve quadratic equations using square roots, completing the square, and the quadratic formula.
- Solve quadratic equations with non-real solutions.
- Use the discriminant to determine the number and type of solutions.
- Solve equations that may include rational expressions or radicals by writing them in quadratic form.
- Use quadratic equations to solve applied problems.
- Solve equations in quadratic form by substitution.
- Solve formulas for variables involving squares and square roots.
- Solve applied problems using the Pythagorean Theorem, area formulas, and quadratic functions as models.
- Solve quadratic inequalities.
- Solve factorable polynomial inequalities.
- Use function operations, including composite functions.
- Find a difference quotient.
- Graph quadratic equations in vertex form using transformations.
- Find a quadratic function to model data.
- Graph quadratic equations in standard form.
- Use quadratic functions to solve problems with minimum or maximum values.
- Graph parabolas with horizontal axes.
- Test for symmetry with respect to the x-axis, y-axis, and the origin.
- Graph piecewise functions.
- Factor polynomials.
- Divide polynomial using synthetic division.
- Find all of the zeros of a polynomial.
- Graph polynomial functions.
- Determine whether a rational function has any vertical, horizontal or oblique asymptotes.

	<ul style="list-style-type: none"> • Graph rational functions.
<p>NJSLS Standards</p>	<p>A.CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A.REI.B.4.a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.</p> <p>A.REI.B.4.b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a + bi$ for real numbers a and b.</p> <p>A.SSE.A.1.a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>A.SSE.A.1.b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.</p> <p>F.BF.B.3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.</p> <p>F.IF.C.7.a. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>N.CN.A.1. Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.</p> <p>N.CN.A.2. Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</p> <p>N.CN.C.7. Solve quadratic equations with real coefficients that have complex solutions.</p> <p>N.CN.C.9. (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.</p> <p>F.IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function</p> <p>F.IF.C.7.b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p>
<p>21st Century Themes Financial and Economic Literacy</p>	<p>21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving</p>

Civic Literacy	Communication and Collaboration
Materials and Resources	Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector
Notes	<ul style="list-style-type: none"> • Chapters 9, 10, & 12 (11 in separate unit) • Exclude solving rational inequalities from Section 9.5. • Skip using the directrix in Section 10.2. • Skip Finding a polynomial function that satisfies given conditions (Complex Zeros) and finding all zeros of a polynomial function given on complex zero in Section 12.2.

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 6: Inverse, Exponential, & Logarithmic Functions
Big Idea/Rationale	<p>In this unit, students will learn methods to solve problems involving unknown exponents by applying an understanding of logarithms. Skills covered will extend previous knowledge from Algebra 2 to focus on more real world application problems in scientific fields such as biology and sociology where you collect organize, and analyze data.</p>
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Identify the relationship between exponential and logarithmic functions as inverses. • Use exponential and logarithmic functions to solve real world problems in science applications such as growth, decay, ion concentration, loudness, and other example. • Logarithms can be used to represent very large or very small numbers because they represent exponents.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • What do you know about a function that is one-to-one? • What is the difference between exponential growth and decay? • Why are the properties of logarithms related to the properties of exponents? • How can missing variables be found in a logarithmic equation?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Terms – one-to-one function, inverse, horizontal line test, exponential function, asymptote, exponential equation, logarithm, logarithmic equation, logarithmic function, common logarithm, natural logarithm, pH, decibel (dB), threshold sound, universal constant, compound interest, continuously compounded interest</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Decide whether a function is one-to-one and, if it is, find its inverse. • Use the horizontal line test to determine whether a function is one-to-one. • Find the equation of the inverse of a function. • Graph f^{-1} from the graph of f. • Define and graph an exponential function. • Solve exponential equations by rewriting with the same base. • Use exponential function in growth and decay applications. • Convert between exponential form and logarithmic form. • Evaluate logarithms. • Graph a logarithmic function. • Use logarithmic functions in growth and decay applications.

	<ul style="list-style-type: none"> • Use the properties of logarithms to write alternative forms of logarithmic expressions. • Evaluate common logarithms and natural logarithms with a calculator. • Use common logarithms and natural logarithms in applications. • Use the change-of-base formula. • Solve exponential and logarithmic equations. • Solve applications of compound interest.
<p>NJSLS Standards</p>	<p>A-CED.A.1. Create equations and inequalities in one variable and use them to solve problems. <i>Include equations arising from linear and quadratic functions, and simple rational and exponential functions.</i></p> <p>A-CED.A.2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p> <p>A-REI.D.11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.</p> <p>F-IF.B.4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. <i>Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</i></p> <p>F-IF.B.5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. <i>For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.</i></p> <p>F-IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>F-IF.C.8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.</p> <p>b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change</p>

	<p>in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.</p> <p>F-LE.1. Distinguish between situations that can be modeled with linear functions and with exponential functions.</p> <p>c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.</p> <p>F-LE.A.4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.</p>
<p>21st Century Themes Global Awareness Financial and Economic Literacy Health Literacy Civic Literacy</p>	<p>21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration</p>
<p>Materials and Resources</p>	<p>Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector</p>
<p>Notes</p>	

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 7: Trigonometric Functions and Their Graphs
Big Idea/Rationale	<p>Students began their study of trigonometry in previous math courses with right triangles and the unit circle. In this unit, they will extend their knowledge to understand how this information can help them solve a variety of real world problems and model periodic phenomena.</p>
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Angles, measured in degrees and radians, can be used to model and solve real world problems. • Trigonometry can be used to measure right triangles as well as oblique triangles. • Real world data can be modeled with a sinusoidal function. • Members within a family of functions, including the trigonometric functions have common characteristics. • Inverse trigonometric functions can be used to solve equations involving angle measures. • Many natural phenomena can be modeled using trigonometric functions. • Trigonometric identities can be used to simplify processes when finding angle measures.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • How can you determine the values of the six trigonometric functions for an angle in standard position? • When an angle is drawn in standard position, how do you know which angle is its reference angle? How do you determine angles that are coterminal with the angle? • What is the advantage of representing an angle measure in radians versus degrees? • Explain the process of evaluating a trigonometric function using reference angles and the unit circle. • Describe the difference between an angle with a positive measure and an angle with negative measure. • What are the reciprocal ratios of sine, cosine and tangent? • How can we use trigonometry to determine the height of a building? • Why are the trigonometric functions periodic? • What are the distinguishing features of the graphs of the sine, cosine, and tangent functions? • What does it mean to find an exact value of a trigonometric function?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Key Terms - initial side, terminal side, complements, supplements, quadrantal angle, coterminal angles, sine, cosine, tangent, cosecant,</p>

	<p>secant, cotangent, cofunctions, reference angle, degrees, minutes, seconds, radian, unit circle, arc length, area of sector, periodic function, period, amplitude, phase shift</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Calculate with degrees, minutes and seconds and convert between DMS and decimal degrees. • Find the measure of coterminal angles. • Review angle relationships and similar triangles. • Find the values of the six trigonometric functions of an angle. • Use identities to find function values. • Find the trigonometric function values of acute angles. • Find a reference angle for a given angle and use it to find the trigonometric function values. • Find function values and angle measures using a calculator. • Solve applied problems involving the angle of elevation or angle of depression. • Convert between degrees and radians. • Find the arc length on a circle and the area of a sector of a circle. • Find exact and approximate circular function values. • Use and draw graphs of trigonometric functions. • Find the amplitude, the period, phase shift, and vertical shift for trigonometric functions. • Write trigonometric equations to model a given situation.
<p>NJSLS Standards</p>	<p>G.SRT.C.6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.</p> <p>G.SRT.C.7. Explain and use the relationship between the sine and cosine of complementary angles.</p> <p>G.SRT.C.8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★</p> <p>F.IF.C.7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★</p> <p>e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>F.TF.A.1. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.</p> <p>F.TF.A.2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.</p>

	<p>F.TF.A.3. (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosines, and tangent for x, $\pi + x$, and $2\pi - x$ in terms of their values for x, where x is any real number.</p> <p>F.TF.A.4. (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.</p> <p>F.TF.B.5. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★</p>
<p>21st Century Themes Global Awareness</p>	<p>21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration</p>
<p>Materials and Resources</p>	<p>Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector</p>
<p>Notes</p>	

Grade: 11th/12th Subject: Advanced Algebra and Trigonometry	Unit 8: A Further Study of Trigonometry
Big Idea/Rationale	Building on their work with trigonometric ratios and functions, students will solve applications involving electricity, displacement, distances that are difficult to measure, areas of oblique triangles and other phenomena.
Enduring Understanding (Mastery Objective)	<ul style="list-style-type: none"> • Identities are important when working with trigonometric functions in calculus. • Trigonometric identities can be used to simplify processes when finding angle measures. • Proving identities gives insight into the way mathematical proofs are constructed. • Equivalent expressions can be written in a variety of formats. • Many natural phenomena can be modeled using trigonometric functions. • Trigonometry can be used to measure right triangles as well as oblique triangles.
Essential Questions (Instructional Objective)	<ul style="list-style-type: none"> • What does it mean to prove an identity? • How can we use an identity to simplify a process involving trigonometric functions? • Can identities be used to find the value of an unknown trigonometric function? • What is the Law of Sines and what is it used for? • What is the Law of Cosines and what is it used for?
Content (Subject Matter)	<p><i>Student will know.....</i></p> <p>Key Terms - identity, fundamental identity, reciprocal identity, quotient identity, Pythagorean identity, odd-even identity (negative-angle identity), verify, sum-difference identity, oblique triangle, Law of Sines, ambiguous case, Law of Cosines, Heron's Formula, semiperimeter</p> <p><i>Student will be able to.....</i></p> <ul style="list-style-type: none"> • Learn and use the fundamental identities. • Verify identities by working with one side. • Use the sum and difference identities for sine, cosine and tangent. • Review the triangle congruence axioms. • Derive the Law of Sines. • Solve AAS and ASA triangles using the Law of Sines. • Uses the SAS formula for the area of a triangle. • Understand the ambiguous case and solve SSA triangles.

	<ul style="list-style-type: none"> Analyze data for the possible number of triangles. Derive the Law of Cosines. Solve SAS and SSS triangles using the Law of Cosines. Use Heron's formula for the area of a triangle.
NJSLS Standards	<p>F.TF.C.8. Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.</p> <p>F.TF.C.9. (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.</p> <p>G.SRT.D.9. (+) Derive the formula $A = 1/2 ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.</p> <p>G.SRT.D.10. (+) Prove the Laws of Sines and Cosines and use them to solve problems.</p> <p>G.SRT.D.11. (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).</p>
21st Century Themes Global Awareness	21st Century Skills Creativity and Innovation Critical Thinking and Problem Solving Communication and Collaboration
Materials and Resources	Algebra and Trigonometry for College Readiness Textbook Graphing Calculator Document Camera/Projector
Notes	