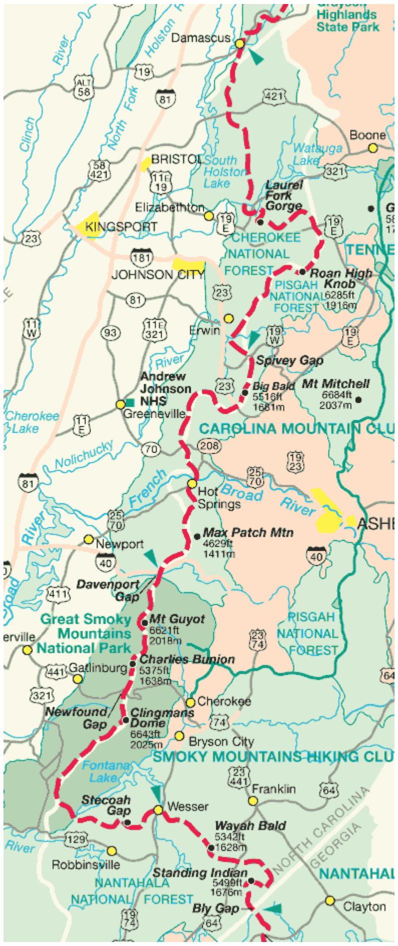
Think About the Situation



The Great Smoky Mountains National Park sits astride the Tennessee-North Carolina border amid the majestic southern climax of the Appalachian Highlands. The most visited of national parks draws more than nine million adventurers and sightseers each year. And for good reason—the Smokies are within a day's drive of a third of the U.S. population, and very few places in the East are in their league as an outdoor-recreation destination.

The Appalachian Trail meanders across the mountain tops of The Great Smoky Mountains National Park. Conceived in 1921 by Benton McKay and initially cleared and marked in 1923, the Appalachian Trail was completed in 1937 and is a marvelous tribute to the well-meaning individuals such as McKay who overcame many obstacles to create this splendid national treasure. The trail winds for 2,015 miles (actually it varies due to changes in sections of the trail) through parts of 14 states. Its southern terminus is Springer, GA and ends (or begins, depending on your perspective) on Mount Katahdin in Maine.

Visitors to the park decided to hike a portion of the trail. When stopping at the first overlook they estimated they had hiked about 2 miles east and 1 mile north.

a) Do you think they hiked directly east and directly north? Explain.
b) How might they have estimated their travel east and north?
c) How far are they from where they started?
d) How far have they hiked?

Investigation: Distance and Midpoint Formulas

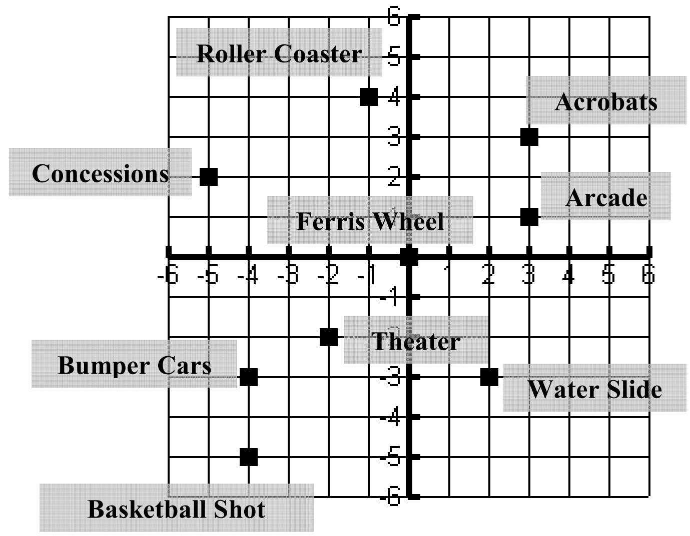
Many hikers rely on Global Positioning Systems (GPS) to determine location and distances; however, there are other methods for determining how far you are from a set point. As you work on the following problems, look for answers to these questions:

What information and calculations are needed to determine distance and location?

1. A coordinate grid can be used to determine how far the visitors are from where they started.
   a. Represent the distance and direction (2 miles east and 1 mile north) by drawing line segments onto a coordinate grid with the starting point at the origin. Label the coordinate point of the overlook where the visitors stopped.
   b. Draw a line segment from the starting point to the overlook. Identify the plane shape that is formed by the line segments drawn on the coordinate grid.
   c. Find the distance between the starting point and the overlook. Show or explain your work.

2. In order to calculate the distance, what information was most important and how did you use the information to calculate the distance?

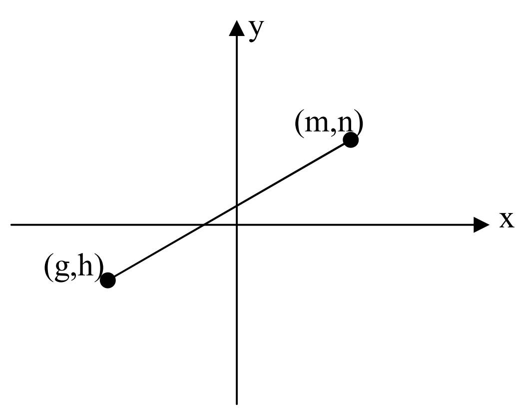
3. The FunFilled Amusement Park is creating a new brochure. They want to include in the brochure distances between some of the most frequently visited attractions. Use the copy of the map and the method describe in activity #2 to find the distances between each pair of attractions.



   a. Ferris Wheel and Arcade
   b. Basketball Shot and the Bumper Cars
   c. Bumper Cars and Water Slide
   d. Roller Coaster and Arcade
   e. Concessions and Theater
   f. Acrobats and Arcade

4. Francine tired of drawing right triangles and looked for a pattern to make finding the distances quicker. She recognized that for each right triangle she used the horizontal distance and the vertical distance between the two points. She decided to investigate a way to get the distances without drawing and counting. Examine Francine's work and help her find a pattern.
   a. First, she looked at the coordinates for the Bumper Cars and the Water Slide since they are on the same horizontal line. Record the coordinates and determine the distance.
   b. How can the distance between the two attractions be determined using the coordinates?
   c. Suppose two points have coordinates (g,h) and (m,h) with g<m.
      i. Explain how you know that the points are on the same horizontal line.
      ii. Write an expression for the distance between the two points.
   d. Next, she looked at the coordinates for the Basketball Shot and the Bumper Cars since they are on the same vertical line. Record the coordinates and determine the distance.
   e. How can the distance between the two attractions be determined using the coordinates?
   f. Suppose two points have coordinates (g,h) and (g,n) with h<n.
      i. Explain how you know that the points are on the same vertical line.
      ii. Write an expression for the distance between the two points.

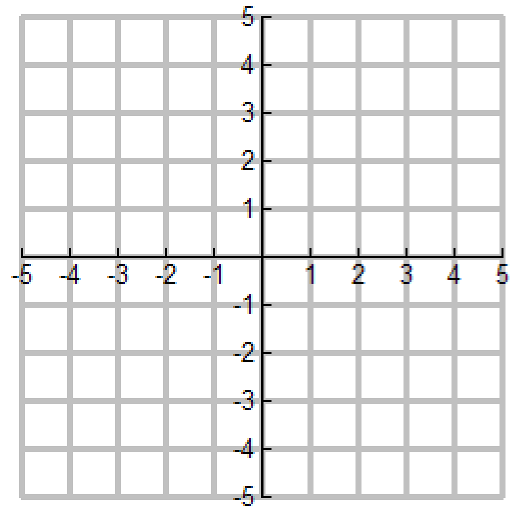
5. Copy the diagram shown to the right and draw the horizontal and vertical segments.
   a. Write expressions for the length of the horizontal segment and the length of the vertical segment.
   b. Recall the Pythagorean Theorem and write an expression for the length of the given line segment.
   c. Use the expression to find the distances between each pair of points.
      i. (3, -2) and (5, -1)
      ii. (2, -1) and (-4, 3)
      iii. (-1, -3) and (4,1)
      iv. (0.5, 2.1) and (4, 2.1)
   d. Compare with others and resolve any differences.



A coordinate grid can also be used to find location. For instance, it can be helpful in estimating the midpoint. This is the point halfway between two points.

6. In the table, pairs of points are given which determine line segments. The point on the segment that is the same distance from each endpoint is called the midpoint. Use the graph to estimate the coordinates of the midpoint for each segment.

Point 1	Point 2	Midpoint
(-3,2)	(1,2)	
(-4,1)	(-2,1)	
(3,3)	(3,-2)	
(1,-3)	(1,2)	
(3,2)	(1,1)	
(-3,-1)	(2,3)	
(4,-2)	(-3,3)	



a. Using the data in the table, search for a pattern that would allow you to determine the coordinates of the midpoints without graphing. Test your conjecture about the midpoints with the pair (4, 9) and (13, 10). Did you get (8.5, 9.5)?
b. Write an expression for the midpoint of a segment with endpoints (g,h) and (m,n).

7. Find the midpoint of each segment with endpoints
a. (3, -2) and (5, -1)
b. (2, -1) and (-4, 3)
c. (-1, -3) and (4,1)
d. (0.5, 2.1) and (4, 2.1)

Summarize the Mathematics

In this investigation, you developed methods for calculating the distance and midpoint of a segment on a coordinate plane. To generalize the methods, consider the general points A(x_1 , y_1) and B(x_2 , y_2)

a) Write a formula for calculating the distance
b) Explain in words how the formula determines the distance.
c) Write a formula for calculating the midpoint
d) Explain in words how the formula determines the midpoint.

Check Your Understanding

On a coordinate grid graph the points A(0,-1), B(2,5), and C(-4, 1)

a) Find the coordinates of the midpoints of $\overline{AB}$ and $\overline{AC}$. Call these midpoints D and E, respectively and plot them on the graph.
b) Compare the lengths of $\overline{DE}$ and $\overline{BC}$.

Investigation: Distance and Midpoints Formulas (Teacher Notes)

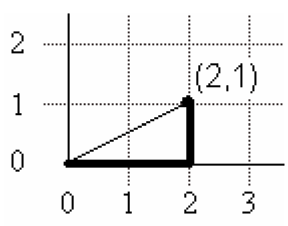
NC CCSS Math 1: Seeing Structure in Expressions
A-SSE.1 Interpret expressions that represent a quantity in terms of its context.*
NC CCSS Math 1: Expressing Geometric Properties with Equations
G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio. *Note: At this level, focus on finding the midpoint of a segment.*
G-GPE.7 Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.

Think About this Situation

a) To travel directly east and directly north is often impossible especially when hiking in the woods.
b) The hikers may have used a map and compass or a GPS tool.
c) Student responses may vary. Possible responses and reasoning may be:
   i. 3 miles by adding 2 miles east and 1 mile north
   ii. less than 3 miles because it would be shorter than going east and north
   iii. more or less than three because the trail could have wound around (be cautious about this response; note that the question is how far are they from where they started)
d) Students may have the same responses as above in part c. Students should discuss that how far they are from where they started and how far they traveled are possibly two different values.

Investigation: Distance and Midpoint Formulas

1.
   a. and b.



The plane shape is a right triangle.

   b. Using the Pythagorean Theorem, $\sqrt{2^2+1^2}=\sqrt{5}\approx 2.2$ units

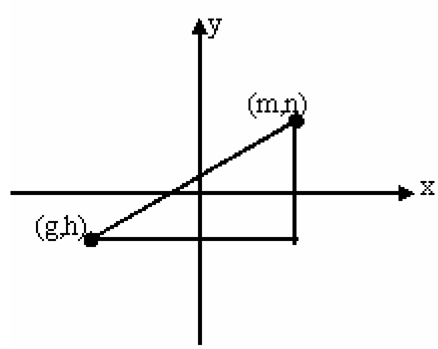
2. The information needed was the horizontal and vertical distances which are the lengths of the legs of the right triangle. The lengths were used with the Pythagorean Theorem to determine the distance from the starting point (origin) to the overlook (2,1).

3. Students may use a combination of counting and use of Pythagorean Theorem to determine distances.
   a. $\sqrt{5}\approx 2.2$ units
   b. 2 units
   c. 6 units
   d. $\sqrt{3^2+4^2}=\sqrt{25}=5$ units
   e. $\sqrt{3^2+4^2}=\sqrt{25}=5$ units
   f. 2 units

4.

a. Bumper Cars (-4,-3) and Water Slide (2,-3) with a distance of 6 units.
b. Subtract the x coordinates
c. Since both points have the same y value of h then the two points are on the same horizontal line. The expression for the horizontal distance is m-g.
d. Basketball Shot (-4,-5) and Bumper Cars (-4,-3) with a distance of 2 units.
e. Subtract the y coordinates.
f. Since both points have the same x value then the two points are on the same vertical line. The expression for the vertical distance is n-h.

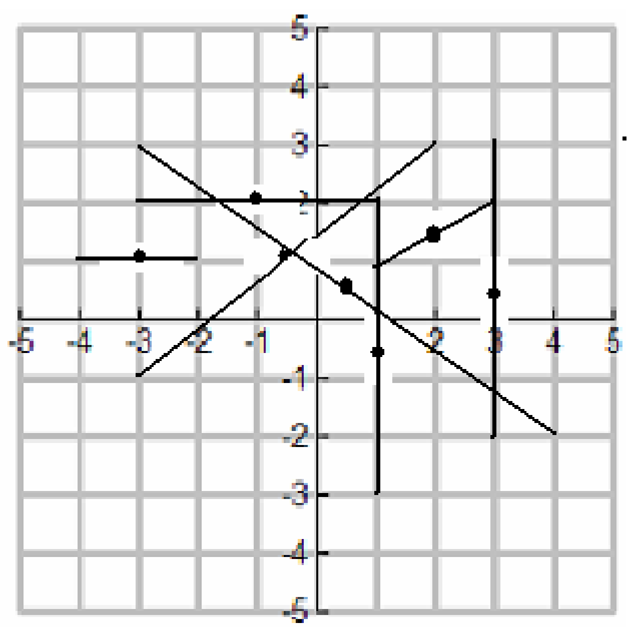
5.



a. The length of the horizontal is m-g and the length of the vertical is n-h
b. $\sqrt{(m-g)^2+(n-h)^2}$
c. Students may use reason in different ways:
    i. horizontal distance 5-3 = 2 units; vertical distance -1-(-2)=1; distance between points $\sqrt{2^2+1^2}=\sqrt{5}$ units
    ii. $\sqrt{(2-(-4))^2+(3-(-1))^2}=\sqrt{52}=2\sqrt{13}\approx 7.2$ units
    iii. $\sqrt{(4-(-1))^2+(1-(-3))^2}=\sqrt{41}\approx 6.4$ units
    iv. The y values are the same so this is a horizontal line segment with a length of 4-0.5=3.5 units.
d. Students should compare strategies and distances.

6.

Point 1	Point 2	Midpoint
(-3,2)	(1,2)	(-1,2)
(-4,1)	(-2,1)	(-3,1)
(3,3)	(3,-2)	(3,0.5)
(1,-3)	(1,2)	(1,-0.5)
(3,2)	(1,1)	(2,1.5)
(-3,-1)	(2,3)	(-0.5,1)
(4,-2)	(-3,3)	(0.5, 0.5)



a. To calculate the midpoint, add the x values and take half and add the y values and take half.
b. $\left(\frac{g+m}{2},\frac{h+n}{2}\right)$

7.

a. (4,-0.5)
b. (-1,1)
c. (1.5,-1)
d. (2.25, 2.1)

Summarize the Mathematics

a) $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$

b) Subtract the x coordinates to determine the horizontal distance and subtract the y coordinates to determine the vertical distance. Finally use the Pythagorean Theorem to add the squared distances and take the square root.

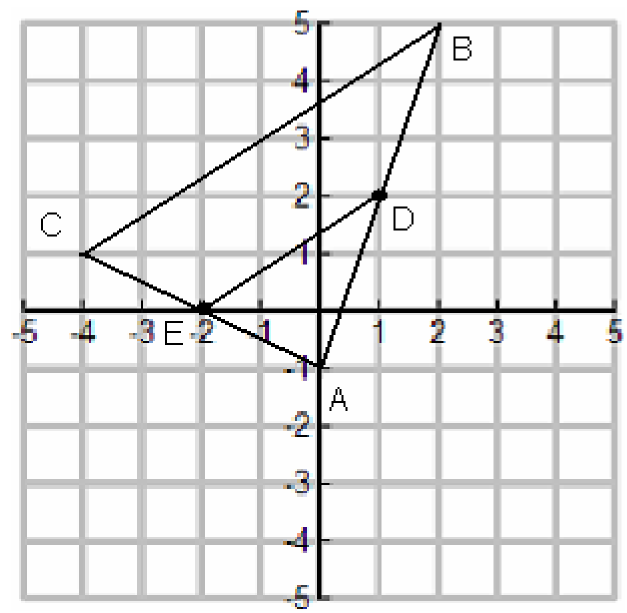
c) $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

d) Add the x coordinates and divide by 2, add the y coordinates and divide by 2

Check Your Understanding

a) D(1,2) and E(-2,0)

b) Length of $\overline{DE}$ is $\sqrt{13} \approx 3.6$ units and the length of $\overline{BC}$ is $\sqrt{52} = 2\sqrt{13} \approx 7.2$ units. The length of $\overline{BC}$ is twice the length of $\overline{DE}$.



Investigation: Distance and Midpoint Formulas

Problem #3

a. Ferris Wheel and Arcade

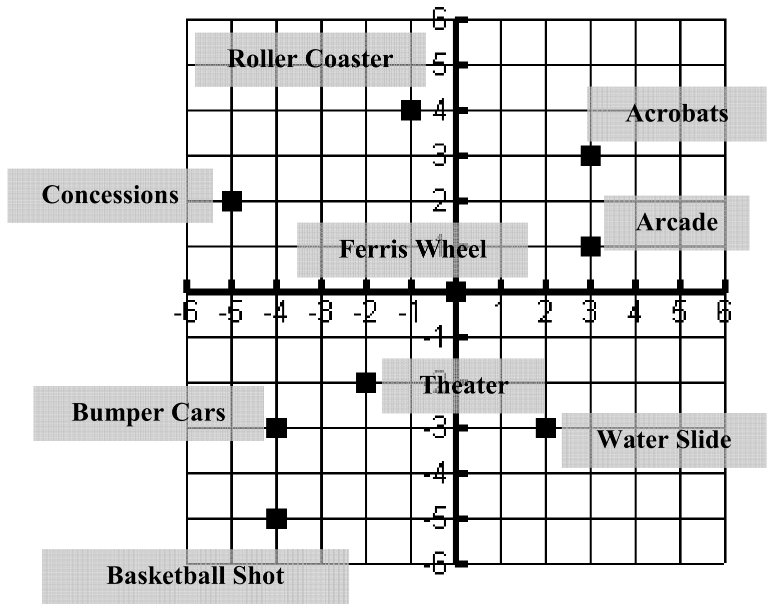
b. Basketball Shot and the Bumper Cars

c. Bumper Cars and Water Slide

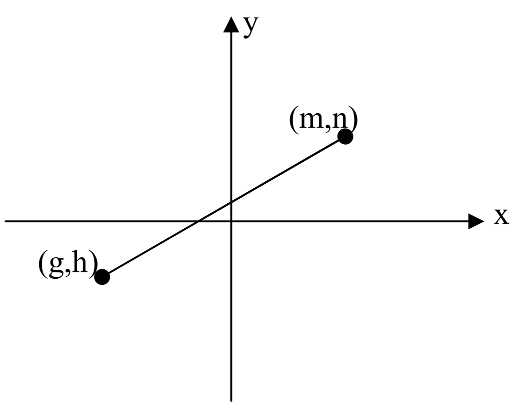
d. Roller Coaster and Arcade

e. Concessions and Theater

f. Acrobats and Arcade



Problem #5



Expression for the length of the horizontal line segment:

Expression for the length of the vertical line segment:

Expression for the length of the given line segment:

Problem #6

Point 1	Point 2	Midpoint
(-3,2)	(1,2)	
(-4,1)	(-2,1)	
(3,3)	(3,-2)	
(1,-3)	(1,2)	
(3,2)	(1,1)	
(-3,-1)	(2,3)	
(4,-2)	(-3,3)