### Question 1

(a) \(2p + 3p^2 = 16 \Rightarrow p = 2\)

**Note:** Do not penalize if \(p = -\frac{8}{3}\) also appears.

(b) \(4xy + 2x^2 \frac{dy}{dx} + 6y \frac{dy}{dx} = 0\)

**Note:** Award (A1) for \(4xy + 2x^2 \frac{dy}{dx}\) and (A1) for \(6y \frac{dy}{dx} = 0\).

At \(P(1, 2)\), \(8 + 2 \frac{dy}{dx} + 12 \frac{dy}{dx} = 0 \Rightarrow 14 \frac{dy}{dx} = -8\)

\[\Rightarrow \text{gradient} = -\frac{4}{7} \approx -0.571\]

### Question 2

(a) \(<0, -1>\)

(b) \(1, 1\)

### Question 3

(a) \(f(g(x)) = x + 1 \Rightarrow [g(x)]^2 = x + 1\)

\[\Rightarrow g(x) = \sqrt{x + 1}\]

(b) \(g(f(x)) = x + 1 \Rightarrow g(x^3) = x + 1\)

\[\Rightarrow g(x) = \sqrt[3]{x + 1}\]

### Question 4

(a) \[\int_{0}^{\infty} \frac{dx}{2x + 3} = \left[ \frac{1}{2} \ln |2x + 3| \right]_{0}^{\infty}\]

\[= \frac{1}{2} \ln \left( \frac{2m + 3}{3} \right) \quad \text{(or} \quad \frac{1}{2} \ln |2m + 3| - \frac{1}{2} \ln 3 \text{)}\]

(b) \[\frac{1}{2} \ln \left( \frac{2m + 3}{3} \right) = 1\]

\[-\frac{2m + 3}{3} = e^2 \left( \frac{2m + 3}{3} = \pm e^2, \text{negative sign not required} \right)\]

\[-\frac{2m + 3}{3} = e^2 \Rightarrow \frac{2m}{3} = e^2 - 1\]

\[m = \frac{3}{2} (e^2 - 1) \approx 9.58\]
### QUESTION 14

(a) Probability
\[ P = 0.2 \times 0.66 + 0.8 \times 0.75 = 0.732 \]

(b) Probability
\[ P(Mon \cap \text{catches train}) = \frac{0.2 \times 0.66}{0.732} = 0.180 \]

### QUESTION 15

(a) 
\[ a = \begin{vmatrix} 1 & 2 & 1 \\ -2 & 0 & 3 \end{vmatrix} = i(6-0) + j(-2-3) + k(0+4) \]
\[ = 6i - 5j + 4k \]

(b) \[ b = -2j + k \]

Length of vector projection is \( l \)
\[ l = \frac{(6i - 5j + 4k) \cdot (-2j + k)}{\sqrt{2^2 + 1^2}} = 14 \]

Unit vector \( \hat{b} = \frac{1}{\sqrt{5}}(-2j + k) \)

Vector projection \[ = l\hat{b} \]
\[ = \frac{14}{\sqrt{5}} \times \frac{1}{\sqrt{5}}(-2j + k) \]
\[ = \frac{-28}{5}j + \frac{14}{5}k \]

### QUESTION 16

Note: If no working shown or if working is incorrect, award (C3) for one correct interval.

METHOD 1
The critical values occur when
\[ x = \pm \frac{23}{9}, \pm \frac{27}{9} \]

Consider: value of function at 0 is 1 which is \( \), 3 ]

Consider: value of function at 12 is 7 which is not \( ]3, 27[ \)

Note: The discontinuity at \( x = 9 \) does not cause any problems since the value of the function is very large in its vicinity.

Consider: value of function at 36 is \( \) which is \( ]27, 3[ \)

The required solution set is therefore \( ]-\infty, 3[ \cup ]27, \infty[ \)

METHOD 2
8 \( f(x) = 3^x \Rightarrow f'(x) = 3^x \ln 3 \)
\( \Rightarrow f''(x) = 3^x (\ln 3)^2 \)
\( 3^x (\ln 3)^2 = 2 \)
\( 3^x = \frac{2}{(\ln 3)^2} \)
\( x \ln 3 = \ln \left( \frac{2}{(\ln 3)^2} \right) \)
\( x = \frac{\ln \left( \frac{2}{(\ln 3)^2} \right)}{\ln 3} \)
\( = 0.460 \)

9
\( \int \frac{\ln x}{\sqrt{x}} \, dx = \int u \frac{dv}{dx} \, dx = \int \ln x d\left( \frac{2x^2}{x} \right) \, dx \)
\( = 2x^2 \ln x - 2 \int \frac{1}{x} \, dx \)
\( = 2x^2 \ln x - 2 \int \frac{1}{\sqrt{x}} \, dx \)
\( = 2x^2 \ln x - 4x^{\frac{3}{2}} + C \)

10 Let \( h \) = height of triangle and \( \theta = \angle CAB \).
Then, \( h = 5 \tan \theta \)
\( \frac{dh}{dt} = 5 \sec^2 \theta \times \frac{d\theta}{dt} \)
Put \( \theta = \frac{\pi}{3} \).
\( 2 = 5 \times 4 \times \frac{d\theta}{dt} \)
\( \frac{d\theta}{dt} = \frac{1}{10} \) rad per sec \( \left( \text{Accept } \frac{18^\circ}{\pi} \text{ per second or } 5.73^\circ \text{ per second} \right) \)