



ALGEBRA II ACTIVITY 16: EXPECTED VALUE

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<p>ACTIVITY OVERVIEW: In this activity we will</p> <ul style="list-style-type: none"> • Use random integer command to simulate rolling a die • Use operations on lists to analyze the probability of rolling the first 1 on the 1st roll, 2nd roll, and so on • Find expected value 	
<p>How many times, in the long run or on average, would you need to roll a die to get a 1? First, use your calculator to experiment and collect some data. Press MATH. Arrow over to the PRB (probability) menu and select 5:randInt(. When you press ENTER this command will be pasted to the home screen. Complete the command as shown.</p>	
<p>Press ENTER. Continue to press it until you “roll” a 1. Stop and record on your paper how many rolls it took. [NOTE: In the sample shown it took 6 rolls. Your experiment will vary.]</p>	
<p>Continue to press ENTER until you “roll” a 1 again. Stop and record on your paper how many rolls it took. [NOTE: In the sample you will have recorded 6 rolls, 1 roll.]</p>	
<p>Repeat. [NOTE: In the sample you will have recorded 6 rolls, 1 roll, 2 rolls.]</p>	

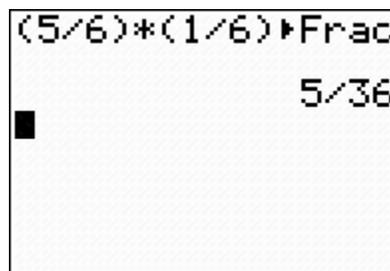
Repeat until you have recorded a total of 10 numbers. [NOTE: In the sample you will have recorded 6 rolls, 1 roll, 2 rolls, 9 rolls.] Find the mean of your 10 numbers. Combine your numbers with your group and calculate the mean. Based on your experiment, how many rolls would you expect to make before a 1 comes up? This is your estimate of the *expected value*, also known as the long-run or mean value.



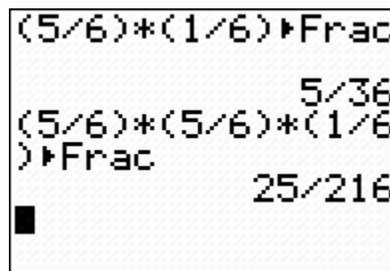
Combine your numbers with your group and calculate the mean. Based on your experiment, how many rolls would you expect to make before a 1 comes up? This is your estimate of the *expected value*, also known as the long-run or mean value.

The probability of rolling a 1 on the first roll is $1/6$. What is the probability of *not* rolling a 1? Using this information, calculate the probability of rolling your first 1 on the second roll.

To use the calculator and get the answer in fraction form, go to the home screen, enter the calculation $(5/6)*(1/6)$ and press **MATH****1****ENTER** to get the answer in fraction form.

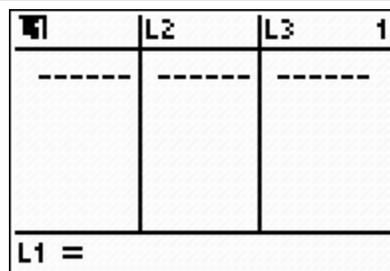


Calculate the probability of rolling your first 1 on the third roll.



Continue for the fourth roll, fifth roll, etc., until a pattern emerges that will help you write a formula for the probability of rolling the first 1 on the n th roll.

Now that you have the formula, analyze the situation further. Press **STAT****ENTER**. Arrow to the top of L1.



Press 2nd STAT , arrow over to **OPS** (operations) menu and select **5:seq**(. When you press ENTER this command will be pasted to the command line for L1. Complete the command **Seq(X,X,1,100,1)**. This will instruct the calculator to fill the list by evaluating the expression X for variable X with values 1 to 100 counting by 1's.

L1	L2	L3	1
-----	-----	-----	
L1 =seq(X,X,1,100,1)			

Arrow up twice to verify that the list has been completed through 100.

L1	L2	L3	1
95			
96			
97			
98			
99			
100			
L1(101)=			

Arrow to the top of L2. Use your formula for the probability of rolling the first 1 on the n th roll. L1 contains the n values, so the formula $5^{(n-1)}/6^n$ will be entered as shown.

L1	L2	L3	2
1	-----	-----	
2			
3			
4			
5			
6			
7			
L2 =5^(L1-1)/6^L1			

Press ENTER . These are the probabilities for rolling the first 1 on the first roll, second roll, third roll, fourth, fifth, and on through the 100th roll. The first few should be the corresponding decimal values for the fractions you found when calculating on the home screen earlier.

L1	L2	L3	2
1	.166667	-----	
2	.13889		
3	.11574		
4	.09645		
5	.08038		
6	.06698		
7	.05582		
L2(1) = .1666666666...			

As you scroll down, how do you think these probabilities will change? Do you think it is very likely that your first 1 would occur on your 25th roll? 75th? 100th?

L1	L2	L3	2
95	6E-9		
96	5E-9		
97	4.2E-9		
98	3.5E-9		
99	2.9E-9		
100	2.4E-9		
L2(101) =			

Arrow to the top of L3. Create a list that is the product of the first two as shown.

L1	L2	L3	3
1	.16667	-----	
2	.13889		
3	.11574		
4	.09645		
5	.08038		
6	.06698		
7	.05582		
L3 =L1*L2			

Press **ENTER**. Press **2nd****MODE** to return to the home screen.

L1	L2	L3	3
1	.16667	.49999	
2	.13889	.27778	
3	.11574	.34722	
4	.09645	.3858	
5	.08038	.40188	
6	.06698	.40188	
7	.05582	.39071	

L3(1)=.166666666...

Press **2nd****STAT**. Right arrow to the **MATH** menu and select **5:sum(**.

NAMES	OPS	▼
1:	min(
2:	max(
3:	mean(
4:	median(
5:	sum(
6:	prod(
7:	stdDev(

Press **2nd****3****□** to complete the command. Press **ENTER**. This sum is the *expected value*. So, over the long-run you would roll a die 6 times to get a 1. How does this sum compare to the estimate you made after your experiment?

sum(L3)	5.99999872
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The *expected value* of a random variable is an average found by multiplying the value of each event by its probability and then summing all the products.