Use what you know about end behavior in order to describe the end behavior of the following polynomial functions. Draw a graph of the function that you would compare the end behavior to.

1. \( f(x) = 2x^7 + 3x^3 - 5x + 1 \)
   
   As \( x \to +\infty \), \( f(x) \to \) ______
   
   As \( x \to -\infty \), \( f(x) \to \) ______

2. \( f(x) = -\frac{1}{2}x^6 + 5x^5 - x^2 + x \)
   
   As \( x \to +\infty \), \( f(x) \to \) ______
   
   As \( x \to -\infty \), \( f(x) \to \) ______

Graph the function. First draw the graph of the function that you would compare the end behavior to. A chart will be needed. Describe the end behavior of the graph of each polynomial function. No calculators!!!

3. \( y = -x^3 + 6x + 1 \)
   
   As \( x \to +\infty \), \( f(x) \to \) ______
   
   As \( x \to -\infty \), \( f(x) \to \) ______

4. Identify whether the function graphed to the right has an even or odd degree and a positive or negative leading coefficient. State the reasons for your choices.

If the equation is a polynomial function, express it in standard form and state its leading coefficient, its degree, and type of polynomial according to its degree. If it is NOT a polynomial function, circle what makes it not one.

5. \( f(x) = \frac{1}{3}x^3 + 5x - x^4\sqrt{2} \)
   
   standard form ____________________________ leading coefficient _________
   
   degree of the polynomial __________ type ______________________________

6. \( f(x) = 3x^2 - \frac{x^5}{4} + 8 \)
   
   standard form ____________________________ leading coefficient _________
   
   degree of the polynomial __________ type ______________________________

7. \( f(x) = 3x^2 - x^{-1} + 8 \)
   
   standard form ____________________________ leading coefficient _________
   
   degree of the polynomial __________ type ______________________________
8. \( f(x) = -8 \)
   standard form ______________________________________________ leading coefficient _________
   degree of the polynomial ______________ type _______________________________________________

9. \( f(x) = 6 - 5x^3 \)
   standard form ______________________________________________ leading coefficient _________
   degree of the polynomial ______________ type _______________________________________________

Perform the indicated operation.

10. \( (6x^3 + 3x^2 - 5x - 1) - (7x^3 + 3x - 6) \)

11. \( (3x - 8)^2 \)

12. \( (2x - 5)(3x^2 + x - 4) \)

13. \( (3x - 7)(x - 2)(3x + 7) \)

14. Consider the expansion of \( (x + 2)^{18} \), how many terms does the expression contain? __________

15. Using Pascal’s triangle and all your vast knowledge, expand each binomial. Remember to simplify all terms. Show all steps and all your work in an orderly, easily read manner.
   \( (2x - 4)^5 \)
FACTOR the polynomials. (LOOK for GCF’s 1st)

16. \(16x^4 - 81\)  
17. \(9x^4 + 18x^3 - 4x^2 - 8x\)

18. \(54x^3 - 16\)

SOLVE each equation by FACTORING. Give all zeros, real and imaginary. When applicable, express your answer in SIMPLIFIED RADICAL form. (LOOK for GCF’s 1st) Do not use the remainder theorem along with synthetic division to do so.

19. \(4x^3 - 20x^2 - 9x + 45 = 0\)  
20. \(x^4 - 4x^2 - 5 = 0\)

21. \(x^4 + 125x = 0\)
Use synthetic substitution to evaluate the following functions.

22. \( f(x) = 4x^4 - 11x^2 + 29x - 22, \ x = -\frac{1}{2} \)  

23. \( f(x) = -x^5 - 5x^3 - 2x^2 + 4x, \ x = 3 \)

\[ f(-\frac{1}{2}) = \underline{\quad} \quad f(3) = \underline{\quad} \]

24. What is another way to evaluate the function? ____________________________________________

25. Use the method from #24 to evaluate \( f(x) = -x^3 - 3x^2 - 8x + 19, \ x = -2. \)  

\( f(-2) = \underline{\quad} \)


\( (4x^3 - 7x + 8) ÷ (2x - 1) \)

27. Divide. Use SYNTHETIC division.

\( (3x^4 + 2x^3 - 5) ÷ (x + 2) \)

28. Which of the following can I easily use synthetic division for? Circle your answer(s).

a. \((x^5 - 2x^3 + x) ÷ (x^2 + 1)\)  
b. \((2x^4 - 3x^3 + x - 8) ÷ (3x - 2)\)  
c. \((7x^2 + 3) ÷ (x - 1)\)

29. Using the rational zero theorem, list all the possible zeros of the function. Do not find the zeros.

\( f(x) = 4x^5 - 8x^4 - 19x^2 + x + 12 \)

30. What does it MEAN to find the zeros of a function? (I do not want to know HOW to find them.)

________________________________________________________________________________________
31. Two zeros of \( f(x) = x^3 - 2x^2 + 9x - 18 \) are \( 3i \) and \( -3i \). Explain why the third zero must be a real number.

Find all the zeros, real and imaginary, of the following functions. Express the answer in simplified radical form when necessary.

<table>
<thead>
<tr>
<th>Function</th>
<th>Prime factored form of the polynomial</th>
<th>Zeros of the function</th>
</tr>
</thead>
<tbody>
<tr>
<td>32. ( f(x) = x^4 + 4x^3 + x^2 - 8x - 6 )</td>
<td>( (x^2 + 2x - 3)(x^2 - 2x - 2) )</td>
<td>( -3, 1 - \sqrt{3}i, 1 + \sqrt{3}i )</td>
</tr>
<tr>
<td>33. ( f(x) = x^3 + 3x^2 - 27x + 7 )</td>
<td>( (x - 7)(x^2 + 10x - 1) )</td>
<td>( 7, -5 - \sqrt{26}i, -5 + \sqrt{26}i )</td>
</tr>
<tr>
<td>34. ( f(x) = 8x^3 - 6x^2 - 23x + 6 )</td>
<td>( (2x - 3)(4x^2 + 4x - 2) )</td>
<td>( \frac{3}{2}, \frac{-1 + \sqrt{3}i}{4}, \frac{-1 - \sqrt{3}i}{4} )</td>
</tr>
<tr>
<td>35. ( f(x) = 2x^3 + 3x^2 + 5x + 2 )</td>
<td>( (x + 1)(2x^2 + x + 2) )</td>
<td>( -1, -\frac{1}{4} - \frac{\sqrt{7}i}{4}, -\frac{1}{4} + \frac{\sqrt{7}i}{4} )</td>
</tr>
</tbody>
</table>
A homeowner is designing a rectangular garden container for her vegetables. The box's width must be 1 meter shorter than its length and its depth must be 4 meters less than its length. The volume of the garden container must be 20 cubic meters.

a. Draw a diagram.
   Then write an equation to model the volume of the container.

b. Find the roots of the equation.

c. Are the roots all rational numbers? Explain.

c. What are the dimensions of the garden container?

Write a polynomial function of least degree that has real number coefficients, the given zeros, and a leading coefficient of one.
Write it in standard form, (for example: \( y = ax^4 + bx^3 + cx^2 + dx + e \))

\(-1, \sqrt{5}, \text{ and } 2i \) BE CAREFUL!

Refer to the function, \( f(x) = 6x^5 + 31x^3 - 64x^2 - 489x - 540 \), in order to answer the questions below.

a. What is the maximum number of turning points ______

b. How many zeros does this same function have? ______

Use technology in order of find the x-intercepts, local maximum(s) and local minimum(s) of the graph of the function. Round your answers to the nearest hundredths when necessary.

\( f(x) = x^4 + x^3 - 5x^2 + 6 \)

x-intercepts _________________________________

local maximum(s) _________________________________

local minimum(s) _________________________________

Can all the zeros of the function, \( f(x) = x^3 - 3x^2 + 9x^2 - 27 \), be found by graphing? Explain your answer.