 MODULE 3  Unit 7 – Sequences

1. Write the first 5 terms of each sequence, then state if it is geometric or arithmetic. How do you know?
   a. \( f(n) = 2^n \)
      \( 2^1, 2^2, 2^3, 2^4, 2^5 \)
      \( 2, 4, 8, 16, 32 \)
      geometric

   b. \( f(n) = 6n + 4 \)
      \( 6(1)+4, 6(2)+4, 6(3)+4, 6(4)+4, 6(5)+4 \)
      \( 10, 16, 22, 28, 34 \)
      arithmetic

   c. \( f(n+1) = f(n) + 3, \ f(1) = 4 \)
      \( 4, 4+3, 7+3, 10+3, 13+3 \)
      \( 7, 10, 13, 16 \)
      arithmetic

   d. \( a_n = 3^n \)
      \( 3^1, 3^2, 3^3, 3^4, 3^5 \)
      \( 3, 9, 27, 81, 243 \)
      geometric

2. Given the following sequences, find \( f(2) \) and state if the sequence is geometric or arithmetic and why.
   a. \( 8, 10, 12, 14, 16 \)
      \( \sqrt{2}, \sqrt{2}, \sqrt{2}, \sqrt{2} \)
      \( f(2) = 10 \)
      arithmetic

   b. \( 3, 6, 12, 24, 48 \)
      \( 3(2), 3(2), 3(2) \)
      \( f(2) = 6 \)
      geometric

   c. \( f(n) = 4^n \)
      \( f(2) = 4^2 \)
      \( f(2) = 16 \)
      geometric

   d. \( f(n) = -2n - 3 \)
      \( f(2) = -2(2) - 3 \)
      \( f(2) = -7 \)
      arithmetic

3. What is the difference between a recursive and an explicit formula? recursive: depends on the # before it
   explicit: \( \frac{4^n}{4} \) rule for sequence, can find any # in the sequence

4. Write an explicit and a recursive formula for the following sequence:
   Term: \( **, ****, ******, ******* \)
   Term #: \( 1, 2, 3, 4 \)
   \( a_n = a_{n-1} + d(n-1) \)
   \( a_n = 2 + 2(n-1) \)
   \( a_n = 2 + 2n - 2 \)
   \( a_n = 2n \)
5. On the accompanying grid, sketch the graphs of \( f(x) = \left(\frac{1}{2}\right)^x \) and \( g(x) = -\frac{3}{2}x + 1 \). Include several key points on each graph and any other key features. Identify the name of the type (family) of each function and the coordinates of all point(s) of intersection.

\[ f(x) = \left(\frac{1}{2}\right)^x \]

Type of function:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( f(x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>2</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1 ( \frac{1}{2} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>2 ( \frac{1}{4} )</td>
<td>( \frac{1}{16} )</td>
</tr>
</tbody>
</table>

\[ g(x) = -\frac{3}{2}x + 1 \]

Type of function:

<table>
<thead>
<tr>
<th>( x )</th>
<th>( g(x) )</th>
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<tbody>
<tr>
<td>-2</td>
<td>4</td>
</tr>
<tr>
<td>-1</td>
<td>3 ( \frac{1}{2} )</td>
</tr>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>1 ( \frac{1}{2} )</td>
<td>( \frac{1}{4} )</td>
</tr>
<tr>
<td>2 ( \frac{1}{4} )</td>
<td>( \frac{1}{16} )</td>
</tr>
</tbody>
</table>

Point(s) of Intersection: \((-2, 4) \neq (0, 1)\)

6. Label the following as linear decay, linear growth, exponential decay, or exponential growth.
   a. Amount of medication remaining in the body over time. \textit{exp. decay}
   b. Population of fish in a pond. \textit{exp. decay}
   c. March Madness playoffs that end in 1 winner. \textit{exp. decay}
   d. Adding $50 to your bank account each month. \textit{linear growth}
   e. \textit{exp. growth}
   f. \textit{linear growth}
   g. \textit{exp. decay}
   h. \textit{exp. linear growth}
   i. \textit{exp. growth}

7. If you are starting a rare stamp collection and you buy 3 new collectable stamps each month, is this situation linear or exponential growth? Write the equation you would use to find how many stamps you would have after \( m \) months.

\textit{linear growth} \hspace{1cm} \ y = 3m
8. If you tell a rumor to 2 people on the first day, and those people each tell 2 more people on the second day, who then tell 2 more people on the 3rd day, does this represent linear or exponential growth? When will more than 50 people know the rumor?

exp. growth

9. Lori's car value decreases by 25% each year. If she bought the care for $3000, after how many years will it be worth less than $1000?

\[ f(t) = 3000(1-.25)^t \]
\[ f(1) = 3000(.75) \]
\[ f(2) = 3000(.75)^2 \]
\[ f(3) = 3000(.75)^3 \]

10. Identify whether each table contains pairs of values that could be modeled by an exponential function, linear function, quadratic function, or none.

a. | X  | Y  |
---|----|
| 0  | 1  |
| 1  | -2 |
| 2  | -5 |
| 3  | -8 |
linear

b. | X  | Y  |
---|----|
| -1 | 1  |
| 0  | 5  |
| 1  | 9  |
| 2  | 13 |
linear

c. | X  | Y  |
---|----|
| -2 | 0  |
| -1 | 1  |
| 0  | 0  |
| 1  | -3 |
quadratic

d. | X  | Y  |
---|----|
| -1 | 2  |
| 0  | 2  |
| 1  | 4  |
exponential

Use the exponential growth and decay formulas to answer the following questions.

Exponential growth: \[ f(t) = A(1+r)^t \]
Exponential decay: \[ f(t) = A(1-r)^t \]

11. In 2000 the population of deer in a local forest was approximately 1,100. If the population decreases at a rate of 4%, write an expression which represents the population five years later.

\[ f(t) = 1100(1-.04)^t \]
\[ f(5) = 1100(.96)^5 \]

12. Joe borrows $500 at 8% interest. Write an equation to represent the amount of money \( f(t) \) that Joe will owe after \( t \) years.

\[ f(t) = 500(1+.08)^t \]
\[ f(t) = 500(1.08)^t \]

13. Mary invests $2000 at .5% interest compounded annually. Write an equation to represent the amount of money \( f(t) \) that Mary will have in the account after \( t \) years.

\[ f(t) = 2000(1+.005)^t \]
\[ f(t) = 2000(1.005)^t \]
14. Which equation models the data in the accompanying table?

<table>
<thead>
<tr>
<th>Time in hours, x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population, y</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>40</td>
<td>80</td>
<td>160</td>
<td>320</td>
</tr>
</tbody>
</table>

a. \( y = 2x + 5 \)  

b. \( y = 2^x \)  

c. \( y = 2x \)  

d. \( y = 5(2)^x \)  

15. \( y = 2^x \)  

Graph

16. \( y = \left( \frac{1}{2} \right)^x \)  

17. \( y = \left( \frac{1}{3} \right)^x \)  

18. \( y = 3^x \)  

19. If the equation \( y = 5^x \) is graphed, which of the following values of x would produce a point closest to the x-axis?

a. \(-3\)  
b. 0  
c. 3  
d. 5  

The correct answer is d. 5.
Use the graph to the right to answer #27-36.

26. Domain \((-\infty, \infty)\)
27. Range \([-2, \infty)\) \(\cup \[0, \infty)\)
28. \(k(-2)\) 6
29. \(k(3)\) 3
30. \(k(0)\) 0
31. \(k(-8)\) -2
32. \(k(-3)\) -2
33. Interval(s) where increasing \((0, \infty)\)
34. Interval(s) where decreasing \((-3, 0)\)
35. Interval(s) where constant \((-\infty, -3)\)

MODULE 4

Units 10 - 12: Quadratics

In #37-45, Factor Completely.

37. \(x^2 - 25\) \((x+5)(x-5)\)
38. \(x^2 - 10x + 24\) \((x-6)(x-4)\)
39. \(6x^2 - 19x + 8\) \((3x-8)(2x-1)\)
40. \(3x^2 - 18x - 21\) \(3(x^2 - 6x - 7)\) \(3(x-7)(x+1)\)
41. \(3xy - 2x^2 + xy^2\) \(xy(3 + 2x + y)\)
42. \(2x^3 y^2 + 6xy^3 + 8xy^2\) \(2xy^2(x^2 + 3y + 4)\)
43. \(y^2 + 49 + 14y\) \((y+7)(y+7)\)
44. \(3k^2 - 2k - 8\) \((3k-4)(k+2)\)
45. \(x^3 - 17x^2 + 72x\) \(x(x^2 - 17x + 72)\) \(x(x-8)(x-9)\)

46. Given the quadratic equation, \(f(x) = x^2 - 4x - 12\), identify the zeros.

\[0 = x^2 - 4x - 12\]
\[0 = (x-6)(x+2)\]
\[x-6=0 \quad x+2=0\]
\[x=6 \quad x=-2\]
47. Given the quadratic equations, write them in vertex form by completing the square, then verify that you did it correctly by putting them back into standard form.

a. \[ y = x^2 + 8x + 6 \]
   \[ y - 6 = x^2 + 8x + \]  
   \[ y - 6 + 16 = x^2 + 8x + 16 \]  
   \[ y + 10 = (x + 4)^2 \]  
   \[ y = (x + 4)^2 - 10 \]  
   \[ \text{Vertex: } (-4, -10) \]

b. \[ f(x) = x^2 - 10x - 2 \]
   \[ y + 2 + \frac{25}{16} = x^2 - 10x + \frac{25}{16} \]  
   \[ y + 27 = (x - 5)^2 \]  
   \[ y = (x - 5)^2 - 27 \]  
   \[ \text{Vertex: } (5, -27) \]

48. Given the quadratic equation \( k(x) = 2(x + 4)^2 - 9 \), identify the translations when compared to the parent graph \( y = x^2 \).

   - vertical stretch: factor of 2
   - horiz. shift: left 4
   - vert. shift: down 9

49. Find the roots of the following quadratic equation, \( (x + 2)^2 = 36 \).

   \[ \sqrt{(x + 2)^2} = \sqrt{36} \]
   \[ x + 2 = \pm 6 \]
   \[ x = -2 \pm 6 \]
   \[ x = 4, -8 \]

50. Given the quadratic equation, \( y = (x - 7)^2 + 8 \), identify the axis of symmetry and vertex.

   AOS: \( x = 7 \)
   Vertex: \( (7, 8) \)

51. Use any method to find the vertex and solutions. \( y = -x^2 - 6x + 16 \).

   Vertex \( (-3, 27) \)
   \[ x = -\frac{b}{2a} = -\frac{-6}{2(-1)} = 3 \]
   \[ x = -3 \]
   \[ y = 27 \]

   Solutions
   \[ 0 = -x^2 - 6x + 16 \]
   \[ 0 = -1(x^2 + 6x - 16) \]
   \[ 0 = -1(x + 8)(x - 2) \]
   \[ x = -8, x = 2 \]
52. Solve \(4x^2 = 20\).
\[
\frac{4}{4} \sqrt{x^2} = \sqrt{5} \\
x = \pm \sqrt{5}
\]

53. How many x-intercepts does \(g(x) = 2x^2 - 4x + 1\) have? How do you know?

\[
\frac{b^2 - 4ac}{16 - 8} \rightarrow 8 \rightarrow 2 \text{ solutions} \\
2 \text{ x-int because positive #}
\]

54. Given the graph, write the function in **vertex form** of the following translation compared to the parent function \(y = x^2\).

**vertex:** \((-3, -4)\)

\[
y = (x+3)^2 - 4
\]

55. Find the axis of symmetry and vertex, and then graph. \(y = (x + 4)(x - 2)\)

**AOS:** \(x = -1\)

\[
y = (-1 + 4)(-1 - 2) \\
y = (3)(-3) \\
y = -9
\]

**Vertex:** \((-1, -9)\)
56. Write a Quadratic Equation that would show a transformation from the parent graph by being shrunk by \( \frac{1}{5} \), shifted 7 to the right, and down 10.

\[ f(x) = \frac{1}{5} (x-7)^2 - 10 \]

57. Write a Quadratic Equation that would show a transformation from the parent graph by being stretched by 4, shifted 2 to the left, and up 3.

\[ y = 4(x+2)^2 + 3 \]

58. Sketch a parabola that shows a vertex of (4,5)

59. Sketch the parabola \( y = x^2 + 6x + 8 \)

\[ y = \frac{-b}{2a} = \frac{-6}{2(1)} = -3 \]

\[ x = -3 \]
\[ (-3)^2 + 6(-3) + 8 \]
\[ 9 - 18 + 8 \]
\[ -9 + 8 \]
\[ -1 \]

\[ v = (-3, -1) \]

60. Describe the transformation from the parent graph of the following Quadratic Equation: \( y = 2(x-4)^2 + 5 \)

- Vertical stretch: factor 2
- Right 4, up 5

Unit 8 – Functions and Transformations

61. Describe the transformations of the absolute value graph described by the equation \( k(x) = -\frac{2}{3}|x-3|-4 \).

- Reflection on x-axis
- Vertical shrink: factor \( \frac{2}{3} \)
- 3 right
- Down 4

62. Write the equation of the absolute value function that stretches by a factor of 2, shifts horizontally to the left 5 and vertically up 9.

\[ y = 2|x+5| + 9 \]
#63-65 - The graph of \( f(x) \) is given to the right. Use it to match each of the transformations to the appropriate equation below. Describe the transformations shown in each function. One equation will not be used. Justify each answer.

\[
\begin{array}{|c|c|}
\hline
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\text{ } & \text{ } \\
\hline
\end{array}
\]

63.

\[h(x)\]

64.

\[g(x)\]

65.

\[k(x)\]
Units 10 – 12

66. Given the graph of the quadratic function below, answer the questions:

![Graph of a quadratic function](image)

Ball Throw

a. What is the domain of the function graphed? \([0,4.5]\)
b. What is the range of the function graphed? \([0,16]\)
c. At what time does the ball hit its maximum? 20 sec
d. What is the maximum height that the ball reaches? 16 feet
e. If the graph continued to the left, where would the other zero be? -5
f. Where is the y-intercept? 6 feet
g. Where is the graph decreasing? \((20,4.5)\)
h. When is the height of the ball about 11 ft? 6 sec & 36 sec

67. Zach throws a hockey puck in the air with an initial velocity of 48 ft/sec from an initial height of 6 feet.

a. Write the equation that represents the height of the puck at time \(t\).

\[
f(t) = -16t^2 + 48t + 6
\]

b. When does the puck hit its maximum height?

\[
x = \frac{-48}{2(-16)} = 1.5 \text{ sec}
\]

c. What is the maximum height of the puck?

\[
y = -16(1.5)^2 + 48(1.5) + 6 = 42 \text{ ft}
\]

d. When does the puck hit the ground?

\[
x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
x = \frac{-48 \pm \sqrt{(48)^2 - 4(-16)(6)}}{2(-16)}
\]

\[
x = \frac{-48 \pm \sqrt{2688}}{-32}
\]

e. What is the y-intercept? What does it represent?

6; Initial height

f. Graph the function,

\[
-48 + \sqrt{2688} = -0.12 \quad -48 - \sqrt{2688} = 3.12
\]
68. If some values of f(x) are shown in the table below, write a new table with the following transformations:

\[
\begin{array}{c|c}
 x_1 & f(x_1) \\
-1 & 2 \\
0 & 0 \\
1 & 2 \\
2 & 5 \\
\end{array}
\]

a) \( g(x) = -f(x) - 3 \)

\[
\begin{array}{c|c|c}
 x & -1 & -2 \\
-1 & -4 & -5 \\
0 & 0 & -3 \\
1 & -2 & -5 \\
2 & -8 & -11 \\
\end{array}
\]

b) \( h(x) = f(x-1) + 2 \)

\[
\begin{array}{c|c|c|c|c|c}
 x & -1 & -2 & 0 & 2 & 0 \\
0 & 1 & 0 & 0 & 2 & 0 \\
1 & 2 & 1 & -2 & 0 & 0 \\
2 & 3 & 2 & -8 & 0 & -6 \\
\end{array}
\]

69. Find the vertex of the following equations, then state how many solutions/roots/x-intercepts they would each have – how do you know?

a) \( f(x) = (x-3)(x+5) \)

\[2 \text{x-int} \rightarrow x = \frac{-(-5)}{2(1)} = 3, -5\]

vertex : \(-1, -16\)

\[x = -1, \quad y = ((-1-3)(-1+5)) = (-4)(4) = -16\]

vertex \((-1, -16)\)

b) \( g(x) = x^2 - 4x + 8 \)

\[x = \frac{-(-4)}{2(1)} = 2\]

\[y = (2)^2 - 4(2) + 8 = 4 - 8 + 8 = 4 + 8\]

vertex \((2, 4)\)

60. Sketch a graph of a quadratic that has:

a) 2 x-intercepts

b) 1 x-intercept

c) 0 x-intercepts