

2.6

Proving Statements about Angles

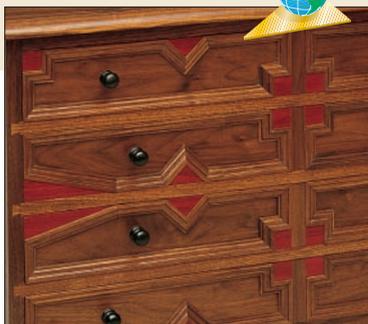
What you should learn

GOAL 1 Use angle congruence properties.

GOAL 2 Prove properties about special pairs of angles.

Why you should learn it

▼ Properties of special pairs of angles help you determine angles in wood-working projects, such as the corners in the piece of furniture below and in the picture frame in Ex. 30.



GOAL 1 CONGRUENCE OF ANGLES

In Lesson 2.5, you proved segment relationships. In this lesson, you will prove statements about angles.

THEOREM

THEOREM 2.2 *Properties of Angle Congruence*

Angle congruence is reflexive, symmetric, and transitive.

Here are some examples.

- REFLEXIVE** For any angle A , $\angle A \cong \angle A$.
- SYMMETRIC** If $\angle A \cong \angle B$, then $\angle B \cong \angle A$.
- TRANSITIVE** If $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\angle A \cong \angle C$.

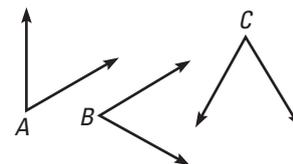
The Transitive Property of Angle Congruence is proven in Example 1. The Reflexive and Symmetric Properties are left for you to prove in Exercises 10 and 11.

EXAMPLE 1 *Transitive Property of Angle Congruence*

Prove the Transitive Property of Congruence for angles.

SOLUTION

To prove the Transitive Property of Congruence for angles, begin by drawing three congruent angles. Label the vertices as A , B , and C .



GIVEN ► $\angle A \cong \angle B$,
 $\angle B \cong \angle C$

PROVE ► $\angle A \cong \angle C$

Statements	Reasons
1. $\angle A \cong \angle B$, $\angle B \cong \angle C$	1. Given
2. $m\angle A = m\angle B$	2. Definition of congruent angles
3. $m\angle B = m\angle C$	3. Definition of congruent angles
4. $m\angle A = m\angle C$	4. Transitive property of equality
5. $\angle A \cong \angle C$	5. Definition of congruent angles

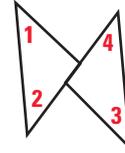


EXAMPLE 2 Using the Transitive Property

This two-column proof uses the Transitive Property.

GIVEN ▶ $m\angle 3 = 40^\circ$, $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$

PROVE ▶ $m\angle 1 = 40^\circ$



Statements	Reasons
1. $m\angle 3 = 40^\circ$, $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 3$	1. Given
2. $\angle 1 \cong \angle 3$	2. Transitive Property of Congruence
3. $m\angle 1 = m\angle 3$	3. Definition of congruent angles
4. $m\angle 1 = 40^\circ$	4. Substitution property of equality

THEOREM

THEOREM 2.3 Right Angle Congruence Theorem

All right angles are congruent.

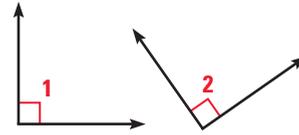


EXAMPLE 3 Proving Theorem 2.3

You can prove Theorem 2.3 as shown.

GIVEN ▶ $\angle 1$ and $\angle 2$ are right angles

PROVE ▶ $\angle 1 \cong \angle 2$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are right angles	1. Given
2. $m\angle 1 = 90^\circ$, $m\angle 2 = 90^\circ$	2. Definition of right angle
3. $m\angle 1 = m\angle 2$	3. Transitive property of equality
4. $\angle 1 \cong \angle 2$	4. Definition of congruent angles

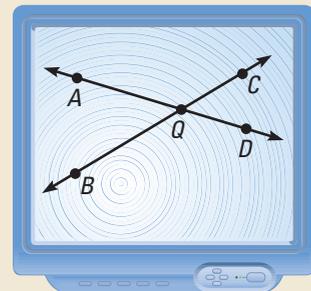
ACTIVITY

Using Technology

Investigating Supplementary Angles

Use geometry software to draw and label two intersecting lines.

- 1 What do you notice about the measures of $\angle AQB$ and $\angle AQC$? $\angle AQC$ and $\angle CQD$? $\angle AQB$ and $\angle CQD$?
- 2 Rotate \overleftrightarrow{BC} to a different position. Do the angles retain the same relationship?
- 3 Make a conjecture about two angles supplementary to the same angle.



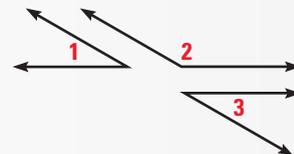
GOAL 2 PROPERTIES OF SPECIAL PAIRS OF ANGLES

THEOREMS

THEOREM 2.4 *Congruent Supplements Theorem*

If two angles are supplementary to the same angle (or to congruent angles) then they are congruent.

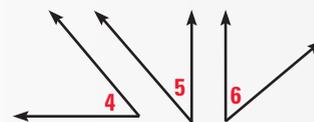
$$\text{If } m\angle 1 + m\angle 2 = 180^\circ \text{ and } m\angle 2 + m\angle 3 = 180^\circ, \text{ then } \angle 1 \cong \angle 3.$$



THEOREM 2.5 *Congruent Complements Theorem*

If two angles are complementary to the same angle (or to congruent angles) then the two angles are congruent.

$$\text{If } m\angle 4 + m\angle 5 = 90^\circ \text{ and } m\angle 5 + m\angle 6 = 90^\circ, \text{ then } \angle 4 \cong \angle 6.$$



Proof

EXAMPLE 4 *Proving Theorem 2.4*

GIVEN $\angle 1$ and $\angle 2$ are supplements,
 $\angle 3$ and $\angle 4$ are supplements,
 $\angle 1 \cong \angle 4$

PROVE $\angle 2 \cong \angle 3$

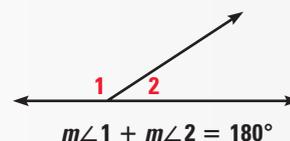


Statements	Reasons
1. $\angle 1$ and $\angle 2$ are supplements, $\angle 3$ and $\angle 4$ are supplements, $\angle 1 \cong \angle 4$	1. Given
2. $m\angle 1 + m\angle 2 = 180^\circ$ $m\angle 3 + m\angle 4 = 180^\circ$	2. Definition of supplementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive property of equality
4. $m\angle 1 = m\angle 4$	4. Definition of congruent angles
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 1$	5. Substitution property of equality
6. $m\angle 2 = m\angle 3$	6. Subtraction property of equality
7. $\angle 2 \cong \angle 3$	7. Definition of congruent angles

POSTULATE

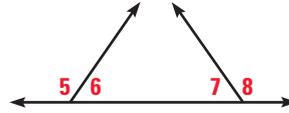
POSTULATE 12 *Linear Pair Postulate*

If two angles form a linear pair, then they are supplementary.



EXAMPLE 5 *Using Linear Pairs*

In the diagram, $m\angle 8 = m\angle 5$ and $m\angle 5 = 125^\circ$. Explain how to show $m\angle 7 = 55^\circ$.



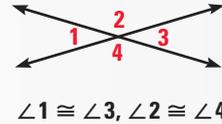
SOLUTION

Using the transitive property of equality, $m\angle 8 = 125^\circ$. The diagram shows $m\angle 7 + m\angle 8 = 180^\circ$. Substitute 125° for $m\angle 8$ to show $m\angle 7 = 55^\circ$.

THEOREM

THEOREM 2.6 *Vertical Angles Theorem*

Vertical angles are congruent.



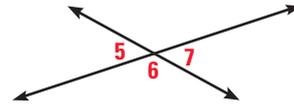
STUDENT HELP

Study Tip
Remember that previously proven theorems can be used as reasons in a proof, as in **Step 3** of the proof at the right.

EXAMPLE 6 *Proving Theorem 2.6*

GIVEN $\angle 5$ and $\angle 6$ are a linear pair,
 $\angle 6$ and $\angle 7$ are a linear pair

PROVE $\angle 5 \cong \angle 7$



Statements	Reasons
1. $\angle 5$ and $\angle 6$ are a linear pair, $\angle 6$ and $\angle 7$ are a linear pair	1. Given
2. $\angle 5$ and $\angle 6$ are supplementary, $\angle 6$ and $\angle 7$ are supplementary	2. Linear Pair Postulate
3. $\angle 5 \cong \angle 7$	3. Congruent Supplements Theorem

GUIDED PRACTICE

Vocabulary Check ✓

1. “If $\angle CDE \cong \underline{\quad? \quad}$ and $\angle QRS \cong \angle XYZ$, then $\angle CDE \cong \angle XYZ$,” is an example of the $\underline{\quad? \quad}$ Property of Angle Congruence.

Concept Check ✓

2. To close the blades of the scissors, you close the handles. Will the angle formed by the blades be the same as the angle formed by the handles? Explain.



Skill Check ✓

3. By the Transitive Property of Congruence, if $\angle A \cong \angle B$ and $\angle B \cong \angle C$, then $\underline{\quad? \quad} \cong \angle C$.

In Exercises 4–9, $\angle 1$ and $\angle 3$ are a linear pair, $\angle 1$ and $\angle 4$ are a linear pair, and $\angle 1$ and $\angle 2$ are vertical angles. Is the statement true?

4. $\angle 1 \cong \angle 3$

5. $\angle 1 \cong \angle 2$

6. $\angle 1 \cong \angle 4$

7. $\angle 3 \cong \angle 2$

8. $\angle 3 \cong \angle 4$

9. $m\angle 2 + m\angle 3 = 180^\circ$

PRACTICE AND APPLICATIONS

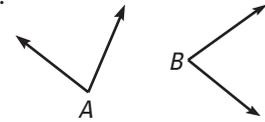
STUDENT HELP

Extra Practice
to help you master
skills is on p. 806.

10. **PROVING THEOREM 2.2** Copy and complete the proof of the Symmetric Property of Congruence for angles.

GIVEN $\angle A \cong \angle B$

PROVE $\angle B \cong \angle A$

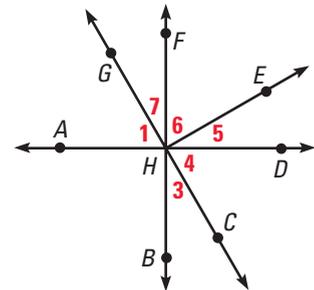


Statements	Reasons
1. $\angle A \cong \angle B$	1. $\underline{\quad ? \quad}$
2. $\underline{\quad ? \quad}$	2. Definition of congruent angles
3. $m\angle B = m\angle A$	3. $\underline{\quad ? \quad}$
4. $\angle B \cong \angle A$	4. $\underline{\quad ? \quad}$

11. **PROVING THEOREM 2.2** Write a two-column proof for the Reflexive Property of Congruence for angles.

FINDING ANGLES In Exercises 12–17, complete the statement given that $m\angle EHC = m\angle DHB = m\angle AHB = 90^\circ$

12. If $m\angle 7 = 28^\circ$, then $m\angle 3 = \underline{\quad ? \quad}$.
 13. If $m\angle EHB = 121^\circ$, then $m\angle 7 = \underline{\quad ? \quad}$.
 14. If $m\angle 3 = 34^\circ$, then $m\angle 5 = \underline{\quad ? \quad}$.
 15. If $m\angle GHB = 158^\circ$, then $m\angle FHC = \underline{\quad ? \quad}$.
 16. If $m\angle 7 = 31^\circ$, then $m\angle 6 = \underline{\quad ? \quad}$.
 17. If $m\angle GHD = 119^\circ$, then $m\angle 4 = \underline{\quad ? \quad}$.



18. **PROVING THEOREM 2.5** Copy and complete the proof of the Congruent Complements Theorem.

GIVEN $\angle 1$ and $\angle 2$ are complements,
 $\angle 3$ and $\angle 4$ are complements,
 $\angle 2 \cong \angle 4$

PROVE $\angle 1 \cong \angle 3$



Statements	Reasons
1. $\angle 1$ and $\angle 2$ are complements, $\angle 3$ and $\angle 4$ are complements, $\angle 2 \cong \angle 4$	1. $\underline{\quad ? \quad}$
2. $\underline{\quad ? \quad}, \underline{\quad ? \quad}$	2. Def. of complementary angles
3. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 4$	3. Transitive property of equality
4. $m\angle 2 = m\angle 4$	4. $\underline{\quad ? \quad}$
5. $m\angle 1 + m\angle 2 = m\angle 3 + m\angle 2$	5. $\underline{\quad ? \quad}$
6. $m\angle 1 = m\angle 3$	6. $\underline{\quad ? \quad}$
7. $\underline{\quad ? \quad}$	7. Definition of congruent angles

STUDENT HELP

HOMEWORK HELP

- Example 1:** Exs. 10, 11
Example 2: Exs. 12–17
Example 3: Exs. 12–17
Example 4: Exs. 19–22
Example 5: Exs. 23–28
Example 6: Exs. 23–28

FINDING CONGRUENT ANGLES Make a sketch using the given information. Then, state all of the pairs of congruent angles.

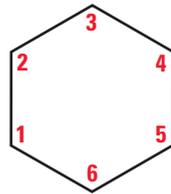
19. $\angle 1$ and $\angle 2$ are a linear pair. $\angle 2$ and $\angle 3$ are a linear pair. $\angle 3$ and $\angle 4$ are a linear pair.
20. $\angle XYZ$ and $\angle VYW$ are vertical angles. $\angle XYZ$ and $\angle ZYW$ are supplementary. $\angle VYW$ and $\angle XYV$ are supplementary.
21. $\angle 1$ and $\angle 3$ are complementary. $\angle 4$ and $\angle 2$ are complementary. $\angle 1$ and $\angle 2$ are vertical angles.
22. $\angle ABC$ and $\angle CBD$ are adjacent, complementary angles. $\angle CBD$ and $\angle DBF$ are adjacent, complementary angles.

WRITING PROOFS Write a two-column proof.

23. **GIVEN** $\triangleright m\angle 3 = 120^\circ, \angle 1 \cong \angle 4,$
 $\angle 3 \cong \angle 4$

PROVE $\triangleright m\angle 1 = 120^\circ$

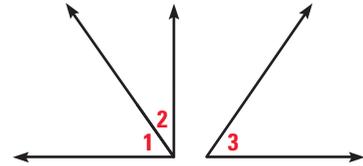
Plan for Proof First show that $\angle 1 \cong \angle 3$. Then use transitivity to show that $m\angle 1 = 120^\circ$.



24. **GIVEN** $\triangleright \angle 3$ and $\angle 2$ are complementary,
 $m\angle 1 + m\angle 2 = 90^\circ$

PROVE $\triangleright \angle 3 \cong \angle 1$

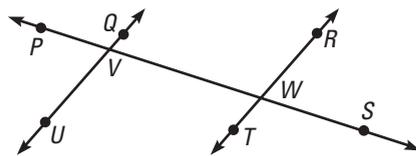
Plan for Proof First show that $\angle 1$ and $\angle 2$ are complementary. Then show that $\angle 3 \cong \angle 1$.



25. **GIVEN** $\triangleright \angle QVW$ and $\angle RWV$ are supplementary

PROVE $\triangleright \angle QVP \cong \angle RWV$

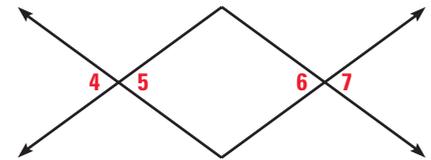
Plan for Proof First show that $\angle QVP$ and $\angle QVW$ are supplementary. Then show that $\angle QVP \cong \angle RWV$.



26. **GIVEN** $\triangleright \angle 5 \cong \angle 6$

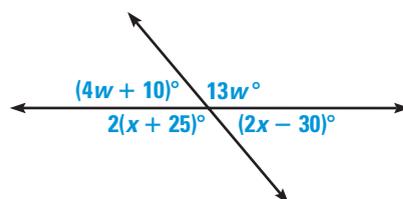
PROVE $\triangleright \angle 4 \cong \angle 7$

Plan for Proof First show that $\angle 4 \cong \angle 5$ and $\angle 6 \cong \angle 7$. Then use transitivity to show that $\angle 4 \cong \angle 7$.

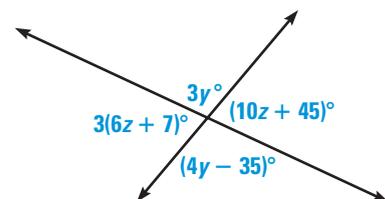


USING ALGEBRA In Exercises 27 and 28, solve for each variable. Explain your reasoning.

27.



28.



STUDENT HELP

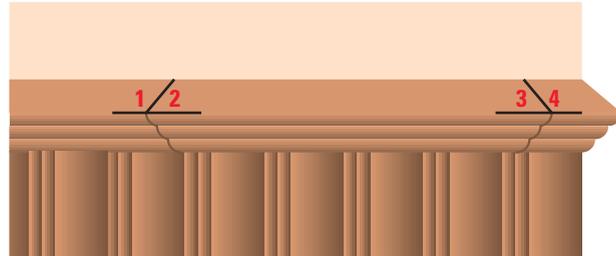
INTERNET
HOMEWORK HELP
 Visit our Web site
www.mcdougallittell.com
 for help with Exs. 23–26.

FOCUS ON APPLICATIONS

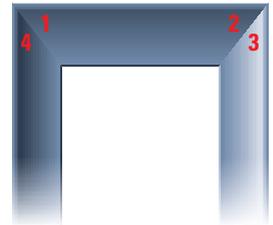


REAL LIFE MITER BOX This box has slotted sides to guide a saw when making angled cuts.

29. **WALL TRIM** A chair rail is a type of wall trim that is placed about three feet above the floor to protect the walls. Part of the chair rail below has been replaced because it was damaged. The edges of the replacement piece were angled for a better fit. In the diagram, $\angle 1$ and $\angle 2$ are supplementary, $\angle 3$ and $\angle 4$ are supplementary, and $\angle 2$ and $\angle 3$ each have measures of 50° . Is $\angle 1 \cong \angle 4$? Explain.



30. **PICTURE FRAMES** Suppose you are making a picture frame, as shown at the right. The corners are all right angles, and $m\angle 1 = m\angle 2 = 52^\circ$. Is $\angle 4 \cong \angle 3$? Explain why or why not.

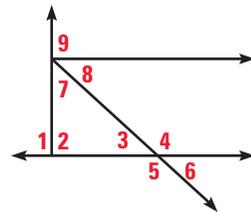


31. **Writing** Describe some instances of mitered, or angled, corners in the real world.
32. **TECHNOLOGY** Use geometry software to draw two overlapping right angles with a common vertex. Observe the measures of the three angles as one right angle is rotated about the other. What theorem does this illustrate?

Test Preparation

QUANTITATIVE COMPARISON Choose the statement that is true about the diagram. In the diagram, $\angle 9$ is a right angle and $m\angle 3 = 42^\circ$.

- (A) The quantity in column A is greater.
- (B) The quantity in column B is greater.
- (C) The two quantities are equal.
- (D) The relationship can't be determined from the given information.



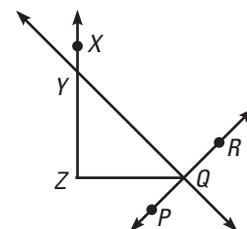
	Column A	Column B
33.	$m\angle 3 + m\angle 4$	$m\angle 1 + m\angle 2$
34.	$m\angle 3 + m\angle 6$	$m\angle 7 + m\angle 8$
35.	$m\angle 5$	$3(m\angle 3)$
36.	$m\angle 7 + m\angle 8$	$m\angle 9$

★ Challenge

37. **PROOF** Write a two-column proof.

GIVEN $\triangleright m\angle ZYQ = 45^\circ$,
 $m\angle ZQP = 45^\circ$

PROVE $\triangleright \angle ZQR \cong \angle XYQ$



MIXED REVIEW

FINDING ANGLE MEASURES In Exercises 38–40, the measure of $\angle 1$ and the relationship of $\angle 1$ to $\angle 2$ is given. Find $m\angle 2$. (Review 1.6 for 3.1)

38. $m\angle 1 = 62^\circ$, complementary to $\angle 2$

39. $m\angle 1 = 8^\circ$, supplementary to $\angle 2$

40. $m\angle 1 = 47^\circ$, complementary to $\angle 2$

41. **PERPENDICULAR LINES** The definition of perpendicular lines states that if two lines are perpendicular, then they intersect to form a right angle. Is the converse true? Explain. (Review 2.2 for 3.1)

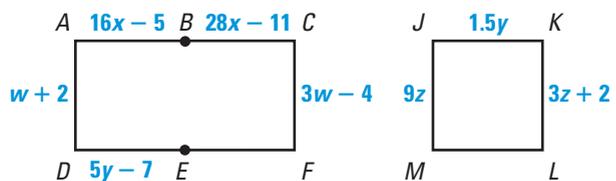
xy **USING ALGEBRA** Use the diagram and the given information to solve for the variable. (Review 2.5)

42. $\overline{AD} \cong \overline{EF}$, $\overline{EF} \cong \overline{CF}$

43. $\overline{AB} \cong \overline{EF}$, $\overline{EF} \cong \overline{BC}$

44. $\overline{DE} \cong \overline{EF}$, $\overline{EF} \cong \overline{JK}$

45. $\overline{JM} \cong \overline{ML}$, $\overline{ML} \cong \overline{KL}$



QUIZ 2

Self-Test for Lessons 2.4–2.6

Solve the equation and state a reason for each step. (Lesson 2.4)

1. $x - 3 = 7$

2. $x + 8 = 27$

3. $2x - 5 = 13$

4. $2x + 20 = 4x - 12$

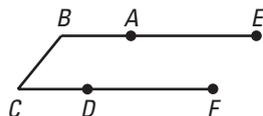
5. $3(3x - 7) = 6$

6. $-2(-2x + 4) = 16$

PROOF In Exercises 7 and 8 write a two column proof. (Lesson 2.5)

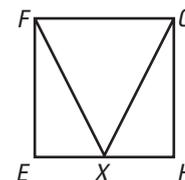
7. **GIVEN** $\triangleright \overline{BA} \cong \overline{BC}$, $\overline{BC} \cong \overline{CD}$,
 $\overline{AE} \cong \overline{DF}$

PROVE $\triangleright \overline{BE} \cong \overline{CF}$



8. **GIVEN** $\triangleright \overline{EH} \cong \overline{GH}$, $\overline{FG} \cong \overline{GH}$

PROVE $\triangleright \overline{FG} \cong \overline{EH}$



9. **ASTRONOMY** While looking through a telescope one night, you begin looking due east. You rotate the telescope straight upward until you spot a comet. The telescope forms a 142° angle with due east, as shown. What is the angle of inclination of the telescope from due west? (Lesson 2.6)

