Name: ____ Date: ________________________

Calculus 2: Summer Assignment

The Fundamental Theorem of Calculus

If a function f is continuous on the closed interval [a,b], and F is an antiderivative of f on the interval [a,b], then $\int_a^b f(x)dx = F(b) - F(a)$.

Examples:

1. $\int_3^7 2x\,dx =$

2. $\int_{-1}^2 \left(6x^2 + 5\right)dx =$

3. $\int_{\frac{5\pi}{6}}^{2\pi} \sin\theta\, d\theta =$

4. $\int_1^e \frac{2}{x}dx =$

5. $\int_2^5 \frac{3x^3 - 2x}{x}dx =$

6. $\int_{\frac{\pi}{4}}^{\frac{\pi}{6}} 4\csc^2 x\,dx =$

U-Substitution

Consider the example: $\int \frac{1}{2} e^{2x}\,dx$

1. Set one part of *f(x)* equal to *u*

2. Take the derivative of both sides of *u*.

3. Substitute *u* and *du* into the integral so the integral is only in terms of *u* and simplify the

<u>Examples: (indefitite integrals)</u>

1. $\int \cos(7x+5)dx$

2. $\int \left(x^2+2x-3\right)^2 \left(x+1\right)dx$

3. $\int 4xe^{x^2}\,dx$

4. $\int 28\left(7x-2\right)^3\,dx$

5. $\int 6x^2\sqrt{x^3+2}\,dx$

6. $\int \frac{9r^2dr}{\sqrt{1-r^3}}$

7. $\int 8(y^4 + 4y^2 + 1)^2(y^3 + 2y)dy$

8. $\int \sin 3x \, dx$

9. $\int x \cos(2x^2)dx$

10. $\int \sec 2x \tan 2x \, dx$

11. $\int (1 - \cos\frac{t}{2})^2 \sin\frac{t}{2} \, dt$

Examples: (definite integrals)

1. $\int_0^3 \sqrt{y+1}dy$

2. $\int_0^1 r\sqrt{1-r^2}dr$

3. $\int_{-\pi/4}^{0} \tan x \sec^2 x dx$

4. $\int_{-1}^{1} \frac{5r}{\left(4+r^2\right)^2} dr$

5. $\int_{0}^{1} \frac{10\sqrt{\theta}}{\left(1+\theta^{3/2}\right)^2} d\theta$

6. $\int_{-\pi}^{\pi} \frac{\cos x}{\sqrt{4+3\sin x}} dx$

7. $\int_{0}^{1} \sqrt{t^5+2t}\left(5t^4+2\right) dt$

8. $\int_{0}^{\pi/6} \cos^{-3} 2\theta \sin 2\theta d\theta$

Integrals of Trigonometric Functions

1. $\int \frac{\csc^2 x}{\cot x}\, dx$

2. $\int (\sec(2x) + \tan(2x))\, dx$

3. $\int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$

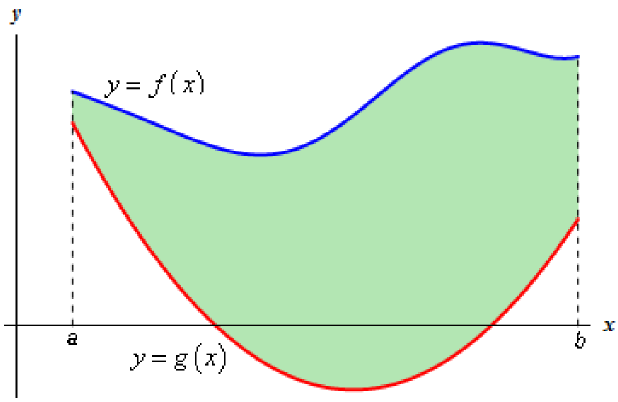
4. $\int_{-\pi/4}^{\pi/4} \frac{\sin^2 x + \cos^2 x}{\cos x}\, dx$

5. $\int_1^2 \frac{1 - \cos x}{x - \sin x}\, dx$

6. $\int_{\pi/4}^{\pi/2} (\csc x - \sin x)\, dx$

Area Between Two Curves

Case 1: $A = \int_a^b f(x) - g(x)dx$

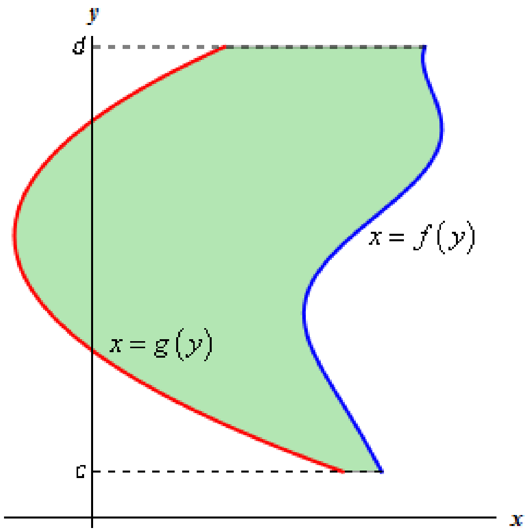


$$A = \int_a^b \begin{pmatrix} \text{upper} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{lower} \\ \text{function} \end{pmatrix} dx, \qquad a \le x \le b$$

Example 1: Determine the area of the region enclosed by $y = x^2$ and $y = \sqrt{x}$

Example 2: Determine the area of the region bounded by $y = xe^{-x^2}$, $y = x + 1$, $x = 2$ and the *y*-axis.

Case 2: $A = \int_c^d f(y) - g(y)dy$



$$A = \int_c^d \begin{pmatrix} \text{right} \\ \text{function} \end{pmatrix} - \begin{pmatrix} \text{left} \\ \text{function} \end{pmatrix} dy, \qquad c \le y \le d$$

Example 3: Determine the area of the region enclosed by $x = \frac{1}{2}y^2 - 3$ and $y = x - 1$.

Example 4: Determine the area of the region bounded by $x = -y^2 + 10$ and $x = (y - 2)^2$.

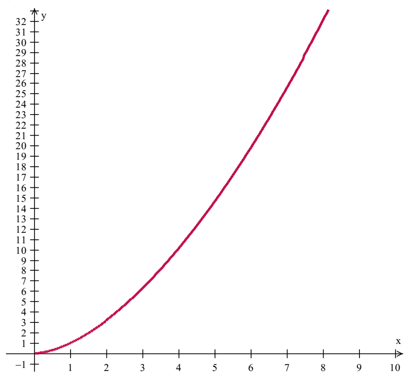
Curves that Intersect at More than two Points

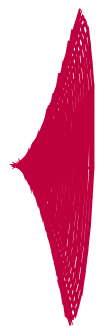
Example 5: Find the area of the region between the graphs of $y = 3x^3 - x^2 - 10x$ and $y = -x^2 + 2x$.

Example 6: Find the area of the region between the graphs of $y = 2x^2 + 10$, $y = 4x + 16$, $x = -2$ and $x = 5$.

Disk Method

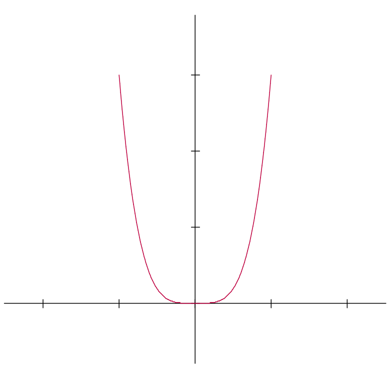
1. The region between the graph of $y = x^{5/3}$, $x = 1$ and $x = 8$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

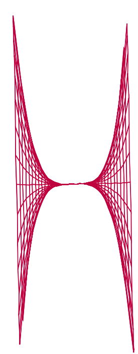




2. Find the volume of the region enclosed by the triangle with vertices (0,1), (0,0) and (1,0) if the region is revolved around the y-axis.

3. Find the volume of the solid generated by revolving the region bounded by the x-axis, the curve $y = 3x^4$, and the lines $x = -1$ and $x = 1$ about the x-axis.





For the following problems find the volume of the solid generated by revolving the region bounded by the lines and curves about the ***x*-axis**. You may use your calculator to graph the functions only.

1. $y = x^2$, $x = 0$, $x = 2$

2. $y = x - x^2$, bounded below by $y = 0$

3. $y = \sqrt{9 - x^2}$, bounded below by $y = 0$

4. $y = \sqrt{\cos x \sin x}$, $x = 0$, $x = \frac{\pi}{2}$

For the following problems find the volume of the solid generated by revolving the region bounded by the lines and curves about the ***y*-axis**. You may use your calculator to graph the functions only.

1. The region enclosed by $x = \sqrt{5}y^2$, $x = 0$, $y = -1$, $y = 1$

2. The region enclosed by $x = y^{3/2}$, $x = 0$, $y = 2$

3. The region enclosed by the triangle with vertices (1,0), (2,1), and (1,1)

4. The region bounded above by the curve $y = \sqrt{x}$ and below by the line $y = x$

Washer Method

1. Determine the volume of the solid obtained by rotating the portion of the region bounded by $y = \sqrt[3]{x}$ and $y = \frac{x}{4}$ that lies in the first quadrant about the y-axis.

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \sqrt{x} + 3$, $y = 1$ and $x = 4$ if the region is revolved about the x-axis.

3. The region in the first quadrant bounded by the graphs of $y = -x^2 + 9$ and $y = \frac{5}{2}x$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

4. Determine the volume of the solid obtained by rotating the region bounded by $y = x^2 - 2x$ and $y = x$, about the line $y = 4$.

5. Determine the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ about the line $x = -1$.